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Evaluation of Carbon Footprints Associated with Cryptocurrency Mining using q-Rung Orthopair Fuzzy Hypersoft Sets

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ABSTRACT

The environmental impact of Bitcoin mining in Kazakhstan, which is currently the third-largest market in the world by hash rate, is coming under further scrutiny. Data on the production of renewable energy and related carbon footprints are essential for evaluating the situation. To create a thorough picture of how Bitcoin mining and environmental responsibility connect in Kazakhstan, this paper allows for the analysis and prediction of the interactions between carbon emissions, renewable energy use, and Bitcoin mining. Using a q-rung orthopair fuzzy hypersoft set (q-ROFHS)-based multi-criteria decision-making technique can improve research on the environmental effects of Bitcoin mining, the integration of renewable energy sources, and the corresponding carbon footprints. The analytic hierarchy process is used to identify the best pollution reduction strategies while taking feasibility and cost-effectiveness into account. The proposed approach will assist the business in achieving its environmental objectives, lessen its negative effects on the environment, and promote a greener future. This study guarantees a more precise and dependable evaluation of pollution control tactics, considering not only the effects on the environment but also practicality and affordability. The outcomes highlight the developed approach's effectiveness and stability in managing complicated information within the parameters of q-ROFHS.

1. Introduction

The impact of cryptocurrencies on the environment has been a topic of concern. Research publications frequently draw attention to the significant carbon footprint associated with mining cryptocurrencies. Significant environmental issues arise from the fact that proof-of-work consensus algorithms, like the one used by Bitcoin, consume a lot of energy. Recent research [1] has found that

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the carbon emissions from cryptocurrency mining are comparable to those of small to medium-sized nations [2]. Increased greenhouse gas emissions occur from the enormous computing power needed for mining operations and the reliance on fossil fuels for electricity generation in some areas. To reduce the global ecological effect of cryptocurrency mining, the industry must shift to more energy-efficient consensus procedures or investigate greener energy sources as the use of cryptocurrencies becomes more widespread. When options, criteria, or parameters are multiple, multi-criteria decision-making (MCDM) is thought to be the viable approach for choosing the best alternatives under several conflicting criteria. Zadeh [3] invented fuzzy sets (FS), which handle uncertainties and deal with ambiguous information in decision-making. FS falls short when experts express membership degrees (MD) in intervals when making decisions. Atanassov [4] developed intuitionistic fuzzy sets (IFS) to address this issue, which Wang and Liu [5,6] expanded with operations and aggregation operators (AOs). There were also created average, geometric, and hybrid AOs [7]. For IFS, new MCDM approaches [8], enhanced similarity measures [9], and basic operations including concentration, dilation, and normalization were also suggested. However, because the present IFS paradigm assumes a linear inequality between MD and non-membership degrees, it struggles to handle contradictory and unclear data. In addition, Yager [10] presented the fundamentally simple orthopair fuzzy sets. Although the methods have a wide range of applications, their usefulness in parametric chemistry is limited by their inefficiency.

Molodtsov developed the soft sets (SS) theory [11], which defined methods for handling uncertainty. Maji et al. [12] invented elementary and binary operations of soft set theory. Fuzzy parametrized SS was first developed by Cagman and Enginoglu [13]. They also expanded its use to decision-making in uncertain situations. SS operations and attributes were refined further by Ali et al. [14]. Fuzzy soft sets (FSS) were created by Maji et al. [15] by combining FS with SS. Roy and Maji [16] developed a unique decision-making approach for FSS to solve the issue of incomplete multi-polar information. AOs for FSS were described by Cagman et al. [17] and a framework for making decisions based on them was offered. Adaptability was added to FSS by Feng et al. [18] and weighted FSS was used in decision-making. Maji et al. [19] proposed intuitionistic fuzzy soft sets (IFSS) and basic operations for additional improvement. IFSS operations were defined in [19,20]. In addition to outlining the fundamental principles of the IFSS, Arora and Gard [21] presented similarity measurements and weighted similarity measures. The concept of an orthopair fuzzy set was presented by Yager [23]. The overview of q-rung orthopair fuzzy sets was proposed by [24], and AOs along with the decision-making technique were presented by Liu and Wang [25]. The idea of q-rung orthopair has become recognized as a useful tool for handling unclear decision-making situations. Q-rung orthopair provides a mathematical framework to represent and handle uncertain information in the field of decision theory, where imprecision and vagueness are inherent in a linguistic environment [26,27]. The q-rung orthopair fuzzy soft sets (q-ROFSS) and related AOs were first introduced by Hussain et al. [28].

The notion of hypersoft sets (HSS) was first presented by Smarandache [29]. HSS is a particularly suited model since it considers parameter sub-attributes and associated ideas like SS. There are several HSS versions, each with unique decision-making processes. The definition of neutrosophic hypersoft sets (NHS) [30], distance and similarity measures of NHSs, and its MCDM techniques along with applications in decision-making problems were proposed by [31-33]. The similarity measures based on NHSs along with the machine learning approach have been proposed by [34]. The concept has been further extended to fuzzy fairly aggregate operators along with material selection application by [35]. Concepts of linguistic hypersoft sets and fuzzy linguistic hypersoft sets were proposed by [36].

Q-rung orthopair fuzzy hypersoft sets (q-ROFHS) were first described by Khan et al. [37] and have simple operations. AOs were proposed by [38,39], and the applications along with the extensions of Einstein operators were proposed by [40,41]. This concept has been further extended to the hybrid intellectual framework known as complex q-ROFHS by [42]. By combining several sources into a single value, the improved structure plays a crucial role in decision-making by addressing uncertainties and data shortages.

i. The work intends to address the environmental effect of energy-intensive mining operations by drawing inspiration from research on carbon emissions in cryptocurrency mining [1,2]. The study investigates the efficacy of various emission reduction techniques based on elements including cost-effectiveness and practicality by using regression analysis, classification algorithms, and clustering techniques.
ii. A thorough comprehension of the methodologies' results is provided using natural language processing tools, which also aid in deriving insightful conclusions from qualitative descriptions.
iii. The urgent need for sustainable solutions is highlighted by the problem of carbon emissions being compounded by the energy-intensive process of Bitcoin mining, which frequently depends on fossil fuels.
iv. This method supports sustainability while reducing negative effects and promoting environmental goals. The results demonstrate the consistency and effectiveness of our method for handling complicated information inside the q-ROFHS's framework.
v. The environmental impact of Bitcoin mining in Kazakhstan, which is currently the third-largest market in the world by hash rate, is coming under further scrutiny. To create a thorough picture of how Bitcoin mining and environmental responsibility connect in Kazakhstan, data on the production of renewable energy and related carbon footprints are essential for evaluating the situation.

Using q-ROFHS in conjunction with MCDM methods can improve research on the environmental effects of Bitcoin mining, the integration of renewable energy sources, and the corresponding carbon footprints. A structured framework for capturing the subtleties of complicated and ambiguous information present in environmental data is offered by q-ROFHS. This paper allows for the analysis and prediction of the interactions between carbon emissions, renewable energy use, and Bitcoin mining. MCDM methods are leveraged in this process. By identifying efficient energy sources, DM tools can optimize strategies for environmentally friendly mining, provide predictive modeling for future carbon footprints based on historical data, and support decision-making systems that assess the viability and cost-effectiveness of sustainability measures. Thus, a potent synergy is created by integrating q-ROFHS with the MCDM technique, allowing for a comprehensive approach to researching and reducing the environmental impact of Bitcoin mining.

The organization of the research paper is structured in the following manner: Section 2 provides the basis of q-ROFHS. In Section 3, we present some fundamental operators and properties. Section 4 offers a well-defined framework for MCDM that utilizes the q-ROFHS-based AHP algorithm. This framework is further illustrated by means of a case study. The findings of the study and their implications are concisely outlined in Section 5, culminating in a discussion of possible avenues for further research.

2. Preliminaries

In this section, we present some necessary definitions, which will be helpful to understand the rest of the paper.

Definition 2.1 [11,12]. Consider $\mathfrak{V}, \breve{\mathcal{U}}$, and $\mathcal{P}(\breve{\mathcal{U}})$ be the set of attributes, universe of discourse, and power set of universes, respectively. Let $\mathfrak{A} \subseteq \mathfrak{E}$ then the pair $(\mathcal{W}, \mathfrak{E})$ is said to be an SS over the universe. Mathematically:

$$\boldsymbol{\mathcal{W}} : \boldsymbol{\mathfrak{V}} \rightarrow \boldsymbol{\mathcal{P}}(\breve{\boldsymbol{\mathcal{U}}}), \tag{1}$$

and defined as:

$$(\boldsymbol{\mathcal{W}}, \boldsymbol{\mathfrak{E}}) = \{\boldsymbol{\mathcal{W}}(\mathbf{e}) \in \boldsymbol{\mathcal{P}}(\breve{\boldsymbol{\mathcal{U}}}) : \boldsymbol{e} \in \boldsymbol{\mathfrak{E}}\}. \tag{2}$$

Definition 2.2 [15]. Consider $\mathfrak{V}, \breve{\mathcal{U}}$, and $\mathcal{P}(\breve{\mathcal{U}})$ be the set of attributes, universe of discourse, and power set of universes, respectively. Let $\mathfrak{A} \subseteq \mathfrak{E}$ then the pair $(\mathcal{W}, \mathfrak{E})$ is said to be an FSS over the universe. Mathematically:

$$\boldsymbol{\mathcal{W}} : \boldsymbol{\mathfrak{V}} \rightarrow \boldsymbol{\mathcal{P}}(\breve{\boldsymbol{\mathcal{U}}}), \tag{3}$$

and defined as:

$$(\boldsymbol{\mathcal{W}}, \boldsymbol{\mathfrak{E}}) = \{\boldsymbol{\mathcal{W}}(\mathbf{e}) \in \boldsymbol{\mathcal{P}}(\breve{\boldsymbol{\mathcal{U}}}) : \boldsymbol{e} \in [\mathbf{0}, \mathbf{1}]\}. \tag{4}$$

Definition 2.3 [29]. The pair $(\mathcal{W}, \mathfrak{H})$ is called an HSS over $\breve{\mathcal{U}}$, where $\mathfrak{H}$ is the cartesian product of *n* disjoint sets $\mathfrak{H}_1, \mathfrak{H}_2, \mathfrak{H}_3, \dots, \mathfrak{H}_\mathrm{p}$ having attribute values of *p* distinct attributes $\mathfrak{H}^1, \mathfrak{H}^2, \mathfrak{H}^3, \dots, \mathfrak{H}^\mathrm{p}$, respectively. Mathematically:

$$\boldsymbol{\mathcal{W}} : \boldsymbol{\mathfrak{H}} \rightarrow \boldsymbol{\mathcal{P}}(\breve{\boldsymbol{\mathcal{U}}}). \tag{5}$$

Definition 2.4 [37]. Consider $\mathfrak{H}, \breve{\mathcal{U}}$, and $\mathcal{P}(\breve{\mathcal{U}})$ be the set of attributes, universe of discourse, and power set of universes, respectively. Let $\mathfrak{H} = \{\mathfrak{H}^1, \mathfrak{H}^2, \mathfrak{H}^3, \dots, \mathfrak{H}^\mathrm{p}\}$, ($p \geq 1$), then assume $\mathrm{q} - \mathrm{ROFHS}$ be a collection of all q-rung orthopair subsets over $\breve{\mathcal{U}}$. Then the pair $(\mathcal{W}, \mathfrak{H}^1 \times \mathfrak{H}^2 \times \mathfrak{H}^3 \times \dots \times \mathfrak{H}^\mathrm{p}) = (\mathcal{W}, \mathfrak{H})$ is known as q-ROFHS. Mathematically:

$$\boldsymbol{\mathcal{W}} : \boldsymbol{\mathfrak{H}}^{\mathbf{1}} \times \boldsymbol{\mathfrak{H}}^{\mathbf{2}} \times \boldsymbol{\mathfrak{H}}^{\mathbf{3}} \times \dots \times \boldsymbol{\mathfrak{H}}^{\mathbf{p}} = \boldsymbol{\mathfrak{H}} \rightarrow \boldsymbol{\mathcal{P}}(\breve{\boldsymbol{\mathcal{U}}}) \ \mathbf{q} - \mathbf{ROFHS}, \tag{6}$$

and defined as $(\mathcal{W}, \mathfrak{H}) = \{(\mathcal{T}_\mathcal{W}(\mathrm{e}), \mathcal{J}_\mathcal{W}(\mathrm{e})) : \mathrm{e} \in \mathfrak{H} \text{ and } (\mathcal{T}_\mathcal{W}(\mathrm{e}), \mathcal{J}_\mathcal{W}(\mathrm{e})) \in [0,1]\}$, where $(\mathcal{T}_\mathcal{W}(\mathrm{e}), \mathcal{J}_\mathcal{W}(\mathrm{e}))$ represent membership and non-membership of attributes, respectively, such that $0 \leq (\mathcal{T}_\mathcal{W}(\mathrm{e}))^\mathrm{q} + (\mathcal{J}_\mathcal{W}(\mathrm{e}))^\mathrm{q} \leq 1$.

3. Aggregation Operators and Properties of q-ROFHSs

In this section, we will discuss the operational laws under q-ROFHSs.

Definition 3.1. Consider $\mathcal{Q} = (\mathcal{T}_{\mathcal{W}}(\mathrm{e}), \mathcal{J}_{\mathcal{W}}(\mathrm{e}))$, $\mathcal{Q}_1 = (\mathcal{T}_{\mathcal{W}1}(\mathrm{e}), \mathcal{J}_{\mathcal{W}1}(\mathrm{e}))$, and $\mathcal{Q}_2 = (\mathcal{T}_{\mathcal{W}2}(\mathrm{e}), \mathcal{J}_{\mathcal{W}2}(\mathrm{e}))$ be three q-ROFHSs, and $\alpha > 0$. Then, the operational laws for q-ROFHSs can be defined as:

1. $\boldsymbol{\mathcal{Q}_1 \oplus \mathcal{Q}_2} = \left\langle \sqrt[q]{(\mathcal{T}_{\mathcal{W}1}(\mathbf{e}))^q + (\mathcal{T}_{\mathcal{W}2}(\mathbf{e}))^q - (\mathcal{T}_{\mathcal{W}1}(\mathbf{e}))^q(\mathcal{T}_{\mathcal{W}2}(\mathbf{e}))^q}, \mathcal{J}_{\mathcal{W}1}(\mathbf{e}).\mathcal{J}_{\mathcal{W}2}(\mathbf{e}) \right\rangle$,
2. $\boldsymbol{\mathcal{Q}_1 \otimes \mathcal{Q}_1} = \left\langle (\mathcal{T}_{\mathcal{W}1}(\mathbf{e}).(\mathcal{T}_{\mathcal{W}2}(\mathbf{e}) \sqrt[q]{\mathcal{J}_{\mathcal{W}1}(\mathbf{e})^q + \mathcal{J}_{\mathcal{W}2}(\mathbf{e})^q - \mathcal{J}_{\mathcal{W}1}(\mathbf{e})^q \mathcal{J}_{\mathcal{W}2}(\mathbf{e})^q} \right\rangle$,
3. $\boldsymbol{\alpha\mathcal{Q}} = \left\langle \sqrt[q]{1 - (1 - \mathcal{T}_{\mathcal{W}}(\mathbf{e})^q)^\alpha}, \mathcal{J}_{\mathcal{W}}(\mathbf{e})^\alpha \right\rangle$,
4. $\boldsymbol{\mathcal{Q}} = \left\langle \mathcal{T}_{\mathcal{W}}(\mathbf{e})^\alpha, \sqrt[q]{1 - (1 - \mathcal{J}_{\mathcal{W}}(\mathbf{e})^q)^\alpha} \right\rangle$.

In the light of the above presented operational laws, we will propose the AOs for the q-ROFHSs environment.

Definition 3.2. Let $\mathcal{Q} = (\mathcal{T}_{\mathcal{W}}(\mathrm{e}), \mathcal{J}_{\mathcal{W}}(\mathrm{e}))$ be a q-ROFHSs, $\mathfrak{w}_i$ be weights of subdivided attributes $\mathfrak{w}_i > 0$, $\sum_{i=1}^{n} \mathfrak{w}_i = 1$. Then, the q-ROFHSWA operator can be:

$$\boldsymbol{q - ROFHSWA}\left(\mathcal{T}_{\mathcal{W}}(\mathbf{e}_{11}), \mathcal{T}_{\mathcal{W}}(\mathbf{e}_{12}), \dots, \mathcal{T}_{\mathcal{W}}(\mathbf{e}_{nm})\right) = \oplus_{i=1}^{n} \mathfrak{w}_i \, \mathcal{Q}. \tag{7}$$

Definition 3.3. Let $\mathcal{Q} = (\mathcal{T}_{\mathcal{W}}(\mathrm{e}), \mathcal{J}_{\mathcal{W}}(\mathrm{e}))$ be a q-ROFHS, $\mathfrak{w}_i$ be weights of subdivided attributes $\mathfrak{w}_i > 0$, $\sum_{i=1}^{n} \mathfrak{w}_i = 1$. Then, the q-ROFHSWGA operator can be:

$$\boldsymbol{q - ROFHSWGA}\left(\mathcal{T}_{\mathcal{W}}(\mathbf{e}_{11}), \mathcal{T}_{\mathcal{W}}(\mathbf{e}_{12}), \dots, \mathcal{T}_{\mathcal{W}}(\mathbf{e}_{nm})\right) = \otimes_{i=1}^{n} \mathcal{Q}^{\mathfrak{w}_i}. \tag{8}$$

Theorem 3.4. Let $\mathcal{Q} = \left(\mathcal{T}_{\mathcal{W}}(\mathrm{e}_{ij}), \mathcal{J}_{\mathcal{W}}(\mathrm{e}_{ij})\right)$ be a q-ROFHSs. Then, the values obtained in Eq. (7) are also q-ROFHSs:

$$\boldsymbol{q - ROFHSWGA}\left(\mathcal{T}_{\mathcal{W}}(\mathbf{e}_{11}), \mathcal{T}_{\mathcal{W}}(\mathbf{e}_{12}), \dots, \mathcal{T}_{\mathcal{W}}(\mathbf{e}_{nm})\right) = \left\langle \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\mathcal{T}_{\mathcal{W}}(\mathbf{e}_{ij})\right)^{\Omega_i}\right), \sqrt[q]{1 - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - \mathcal{J}_{\mathcal{W}}(\mathbf{e}_{ij})^q\right)^{\Omega_i}\right)} \right\rangle, \tag{9}$$

where $\mathfrak{w}_i$ are weights for sub-divided attributes with $\mathfrak{w}_i > 0$, $\sum_{i=1}^{n} \mathfrak{w}_i = 1$.

Example 3.5. Let $D = \{d_1, d_2, d_3\}$ represents the set of decision-makers with weights $\mathfrak{w}_i = (0.15, 0.52, 0.33)^T$. The decision-makers will assign values to the set of attributes of a small beach-facing house based on their expertise and experience. Set of attributes $\mathfrak{H} = \{\mathfrak{H}_1 = \text{terrace}, \mathfrak{H}_2 = \text{security system}\}$ with their corresponding sub-attributes "terrace" $= \mathfrak{H}_1 = \{\mathfrak{H}_{11} = \text{single}, \mathfrak{H}_{12} = \text{double}\}$ and "security system" $= \mathfrak{H}_2 = \{\mathfrak{H}_{21} = \text{guards}, \mathfrak{H}_{22} = \text{cameras}\}$. Let $\mathfrak{H} = \mathfrak{H}_1 \times \mathfrak{H}_2 = \{(\mathfrak{H}_{11}, \mathfrak{H}_{21}), (\mathfrak{H}_{11}, \mathfrak{H}_{22}), (\mathfrak{H}_{12}, \mathfrak{H}_{21}), (\mathfrak{H}_{12}, \mathfrak{H}_{22})\}$ be a set of sub-attributes. Decision-makers will make a mathematical model in the form of q-ROFHSs for each multi-sub-attribute, and they will assign values:

$$(\mathcal{W}, \mathfrak{H}) = \begin{bmatrix} (0.3, 0.8) & (0.4, 0.6) & (0.3, 0.6) & (0.5, 0.6) \\ (0.8, 0.3) & (0.7, 0.4) & (0.7, 0.3) & (0.4, 0.8) \\ (0.3, 0.6) & (0.5, 0.7) & (0.6, 0.5) & (0.5, 0.4) \end{bmatrix}.$$

Using Eq. (9):

$$q-ROFHSWGA\left(\mathcal{T}_{\mathcal{W}}(\mathrm{e}_{11}),\mathcal{T}_{\mathcal{W}}(\mathrm{e}_{12}),\ldots,\mathcal{T}_{\mathcal{W}}(\mathrm{e}_{34})\right) =$$

$$\left\langle \prod_{j=1}^{4}\left(\prod_{i=1}^{3}\left(\mathcal{T}_{\breve{d}_{ij}}\right)^{\Omega_i}\right), \sqrt[q]{1-\prod_{j=1}^{4}\left(\prod_{i=1}^{3}\left(1-\mathcal{J}_{\breve{d}_{ij}}{}^{q}\right)^{\Omega_i}\right)}\right\rangle$$

$$= \left\langle \begin{array}{c} \left(\begin{array}{c}\{(.3)^{.143}(.8)^{.514}(.3)^{.343}\}^{.35}\{(.4)^{.143}(.7)^{.514}(.5)^{.343}\}^{.15}\{(.3)^{.143}(.7)^{.514}(.6)^{.343}\}^{.2}\\ \{(.4)^{.143}(.6)^{.514}(.4)^{.343}\}^{.3}\end{array}\right), \\ \sqrt{1-\left(\begin{array}{c}\{(.36)^{.143}(.91)^{.514}(.64)^{.343}\}^{.35}\{(.64)^{.143}(.84)^{.514}(.51)^{.343}\}^{.15}\{(.64)^{.143}(.91)^{.514}(.75)^{.343}\}^{.2}\\ \{(.75)^{.143}(.84)^{.514}(.75)^{.343}\}^{.3}\end{array}\right)} \end{array}\right\rangle$$

$= \langle 0.4679, 0.5590\rangle$.

Definition 3.6. Properties of q-ROFHSs are:

(i) *Boundedness* – Let $\mathcal{Q}_i$ be a collection of q-ROFHSs and $\mathcal{T}_{\mathcal{W}}(\mathrm{e}_{ij})^{-} = \left\langle \min_j \min_i \{\mathcal{T}_{\mathcal{W}}(\mathrm{e}_{ij})\}, \max_j \max_i \{\mathcal{J}_{\mathcal{W}}(\mathrm{e}_{ij})\}\right\rangle$, and $\mathcal{T}_{\mathcal{W}}(\mathrm{e}_{ij})^{+} = \left\langle \max_j \max_i \{\mathcal{T}_{\mathcal{W}}(\mathrm{e}_{ij})\}, \min_j \min_i \{\mathcal{J}_{\mathcal{W}}(\mathrm{e}_{ij})\}\right\rangle$, then $\mathcal{T}_{\mathcal{W}}(\mathrm{e}_{ij})^{-} \leq \mathcal{T}_{\mathcal{W}}(\mathrm{e}_{ij})^{+}$.

(ii) *Idempotency* – $\mathcal{Q} = \left(\mathcal{T}_{\mathcal{W}}(\mathrm{e}_{ij}), \mathcal{J}_{\mathcal{W}}(\mathrm{e}_{ij})\right)$, $\forall i,j$, then $\left(\mathcal{T}_{\mathcal{W}}(\mathrm{e}_{11}),\mathcal{T}_{\mathcal{W}}(\mathrm{e}_{12}),\ldots,\mathcal{T}_{\mathcal{W}}(\mathrm{e}_{nm})\right) = \mathcal{T}_{\mathcal{W}}(\mathrm{e}_{ij})$.

(iii) *Homogeneity* – $\left(\alpha\mathcal{T}_{\mathcal{W}}(\mathrm{e}_{11}),\alpha\,\mathcal{T}_{\mathcal{W}}(\mathrm{e}_{12}),\ldots,\alpha\mathcal{T}_{\mathcal{W}}(\mathrm{e}_{nm})\right) = \alpha\mathcal{T}_{\mathcal{W}}(\mathrm{e}_{ij})$ for any $\alpha > 0$.

4. q-ROFHS-based AHP Algorithm

In this section, we present an MCDM that utilizes the basics of q-ROFHSs. This algorithm is further illustrated by solving a case study of carbon footprints associated with Bitcoin mining.

4.1 Algorithm

The prominent MCDM technique is used by XYZ manufacturing company to assess the options and criteria. The AHP entails comparing alternatives in pairs according to the relative weight of each criterion. The best option is then determined by synthesizing these comparisons using ML tools. The implementation procedure is presented below.

i. *Pairwise comparisons of criteria* – The company's decision-makers give each criterion a relative weight according to its significance. For example, if "emission reduction potential" is considered more significant than "employee engagement", it would be given a larger weight.
ii. *Alternative pairwise comparisons* – Based on how each criterion contributes to the overall objective, comparable pairwise comparisons are conducted for the alternatives.
iii. *Weighted score calculation* – Each option and criterion combination's weighted scores are determined using the relative weights.
iv. *Synthesis of results* – To ascertain the most advantageous carbon footprint reduction plan for XYZ Manufacturing Company, the weighted scores are combined.

The stepwise procedure of the algorithm is presented Table 1. The pictorial representation of the flowchart is presented in Figure 1.

Step 1. Consider goal ($\mathcal{G}$), criteria's ($\mathcal{C}_i$), and alternative's $\mathcal{A}_j$.

Step 2. Make a $\mathcal{W}^l = [\mathcal{W}_{ij}]$ pairwise comparison matrix (e.g., criteria or alternatives).

Step 3. Calculate the weighted normalized matrix $\mathcal{N}_{ij}^{l} = [\mathcal{W}_{ij}] \times \omega_j$.

Step 4. Calculate a weighted sum $\mathcal{S}_j = \sum_{j=1}^{k}\mathcal{N}_{ij}^{l}$ for each criterion ($\mathcal{C}_i$).

Step 5. Calculate the consistency ratio and analyze the ranking.

Table 1
q-ROFHS-based AHP algorithm

Input Goal, Criteria's $\mathcal{C}_i$, and Alternative's $\mathcal{A}_j$	
Output Ranking and final prioritization	
1	**for** $\mathcal{A}_j$ $\leftarrow$ 1 to n
	for $\mathcal{C}_i \leftarrow$ 1 to m
2	**Read** [$\mathcal{C}_i$ criteria for each alternative depending on DM choice]
3	**for** DM $\leftarrow$ p
4	**for** $\mathcal{C}_i \leftarrow$ m
5	**for** $\mathcal{A}_j$ $\leftarrow$ n
6	**Output** $\leftarrow [\mathcal{W}_{ij}]$
	endfor
7	**for** $\omega_j \leftarrow$ n
8	**Output** $\leftarrow \mathcal{N}_{ij}^{l} = [\mathcal{W}_{ij}] \times \omega_j$.
	endfor
9	**for** $\mathcal{S}_j \leftarrow$ n
10	**Output** $\leftarrow \mathcal{S}_j = \sum_{j=1}^{k} \mathcal{N}_{ij}^{l}$
	endfor
11	**Ouput** Rank or goal $\boldsymbol{R_p}$ $[\boldsymbol{R_p} : p = 1,2, \dots, n]$
12	**Read** R_p
13	**Output =** Rank the alternatives
	endfor

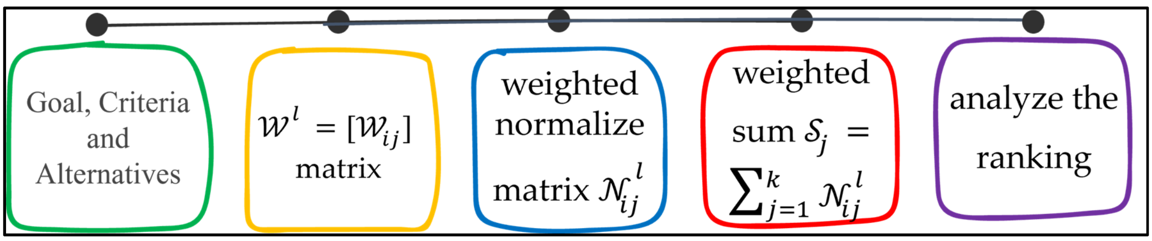

Fig. 1. The flowchart of the proposed algorithm

4.2 Case Study

Considering the increasing environmental risks associated with the rapidly expanding cryptocurrency market, case studies are crucial. Due to the energy-intensive mining process, the demand for digital currencies, especially Bitcoin, increases along with energy consumption and carbon emissions. This study aims to determine the amount of carbon footprints generated by Bitcoin mining operations and to evaluate and quantify such effects. Stakeholders can implement initiatives that promote sustainability, lessen environmental impact, and contribute to the responsible expansion of the Bitcoin sector by having a clear awareness of the ecological effects.

This case study's benefits come from its ability to offer practical advice for reducing the negative effects of cryptocurrency mining on the environment. Effective pollution reduction solutions can be identified by the research through thorough data analysis and the application of decision-making tools like AHP. Furthermore, the research endeavors to suggest workable and feasible alternatives by considering environmental impact in addition to feasibility and cost-effectiveness. It is important to recognize the limits that come with these kinds of assessments, though. These could include the difficulty of forecasting future energy patterns, the dynamic nature of cryptocurrency markets and technologies, and concerns regarding the veracity of data. Notwithstanding these drawbacks, the research represents a ground-breaking attempt to strike a balance between the expansion of cryptocurrencies and environmental stewardship, enabling participants to embrace sustainable behaviors in a dynamic digital environment.

An estimated 348 terawatt hours of electricity are used annually by Bitcoin miners. The renewable proportions for the United States, China, and Kazakhstan were, on a national level, 22.5%, 30.2%, and 11.3%, respectively. To put things in perspective, excluding nuclear power, 30% of the world's electricity was generated by renewable sources in 2022. Kazakhstan's pitiful renewable share results from its 60% reliance on coal, a valuable export from the central Asian nation. In China, coal accounts for 61% of total electricity production, but due to the country's rapid growth in wind and solar power, the country's overall renewable energy proportion is greater.

The environmental impact of Bitcoin mining in Kazakhstan, which is currently the third-largest market in the world by hash rate, is coming under further scrutiny. Data on the production of renewable energy and related carbon footprints are essential for evaluating the situation. To create a thorough picture of how Bitcoin mining and environmental responsibility connect in Kazakhstan. Using q-ROFHS in conjunction with decision-making methods can improve research on the environmental effects of Bitcoin mining, the integration of renewable energy sources, and the corresponding carbon footprints. A structured framework for capturing the subtleties of complicated and ambiguous information present in environmental data is offered by q-ROFHS. This framework allows for the analysis and prediction of the interactions between carbon emissions, renewable energy use, and Bitcoin mining.

On a life-cycle basis, renewable energy produces between 11 and 740 gCO2 for every kWh produced, depending on the kind (solar, wind, hydro, geothermal, tidal, wave, biomass) [2]. The mining process of Bitcoin is associated with energy and blockchain networks. The amount of energy the network uses will depend on the hash rate of the entire Bitcoin network. The hash rate of a blockchain network increases with the number of computers that connect to it and process hashes (guesses) on the network. A PoW blockchain network with a high hash rate is more secure and healthy since there is less likelihood of an attack. Energy use will decrease with a lower network hash rate. The network will need more energy to mine each new block when the hash rate is higher. BTC is produced using 2.7 quadrillion computed hashes. The production of one Bitcoin can consume 663.68kWh of energy and it produces 370.17 kgCO2 [4,5]. Through the potential for financial gain, job development, infrastructure investment, and technical advancement, Bitcoin mining may improve the economy. Bitcoins may be earned as rewards, and miners can also invest in cutting-edge gear and data centers, stabilize the energy markets, improve technology, and promote financial inclusion (Figure 2).

Step 1: Consider five renewable energy resources $\mathcal{R}^1$ (hydrogen), $\mathcal{R}^2$ (wind and hydro), $\mathcal{R}^3$ (solar), $\mathcal{R}^4$ (geothermal), and $\mathcal{R}^5$ (nuclear) as alternatives $\mathcal{R} = \{\mathcal{R}^1, \mathcal{R}^2, \mathcal{R}^3, \mathcal{R}^4, \mathcal{R}^5\}$. Kazakhstan has the potential to commercially gain from the cryptocurrency business, and with this study, we want to determine which renewable energy should be used to increase the Bitcoin mining process that meets its economic targets. The services of the experts in this domain were taken as decision-makers $\mathcal{D} = \{\mathcal{D}^{\mathrm{m}}\ ; m = 1,2\}$. Consider the parameters: $\mathcal{P}^1 = \text{cost of production}$, $\mathcal{P}^2 = \text{Carbon footprints}$, and $\mathcal{P}^3 =$ economic benefits. Their respective parametric values are:

i. $\text{Cost of production} - \mathcal{P}^1 = \{< \$20/MWh, < \$40/MWh, < \$80/MWh, < \$100/MWh\}$,
ii. Carbon footprints $- \mathcal{P}^2 = \{10 - 200\ gCO_2,\ 201 - 400\ gCO_2,\ 401 - 600\ gCO_2,\ 601 - 800\ gCO_2\}$,

iii. Economic benefits (per day) – $\mathcal{P}^3 = \{100\ BTC, 1000\ BTC, 10000\ BTC\}$.

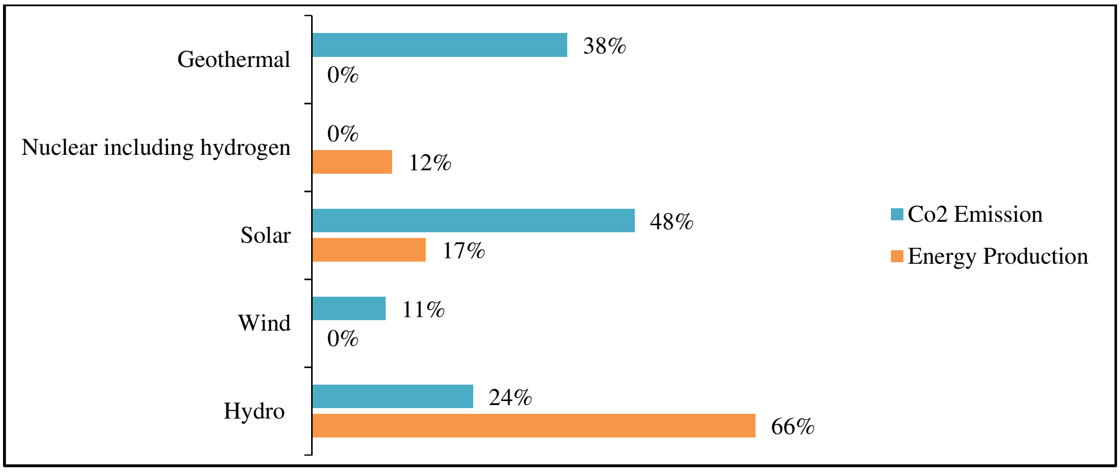

Fig. 2. CO2 emission associated with different types of energy production

Then, the function $\Gamma: \Lambda = \mathcal{P}^1 \times \mathcal{P}^2 \times \mathcal{P}^3 \longrightarrow P(\Omega)$, where $M = \{\mathcal{R}^1, \mathcal{R}^2, \mathcal{R}^3, \mathcal{R}^4, \mathcal{R}^5\} \subset \Omega$, with $\Omega = \mathcal{R}$ as the universal set. The sub-divided parametric function can be given below:
$\Gamma(\$20/MWh, 140 - 400\ gCO_2,\ 100BTC) = \{\mathcal{R}^1, \mathcal{R}^2, \mathcal{R}^3, \mathcal{R}^4, \mathcal{R}^5\}$.

Step 2. Make a $\mathcal{W}^l = [\mathcal{W}_{ij}]$ pairwise comparison matrix (e.g. criteria or alternatives).
$\mathcal{D}^1$ assigned q − ROFHSs values to the parametric choices as:

$$\mathcal{W}^1 = \left\{\begin{array}{l} \mathcal{R}^1 < \dfrac{\$20\ per\ MWh}{(0.4,0.6)}, \dfrac{140 - 400\ gCO_2}{(0.9,0.7)}, \dfrac{100BTC}{(0.5,0.8)} >, \\ \mathcal{R}^2 < \dfrac{\$20\ per\ MWh}{(0.33,0.2)}, \dfrac{140 - 400\ gCO_2}{(0.12,0.27)}, \dfrac{100BTC}{(0.31,0.2)} >, \\ \mathcal{R}^3 < \dfrac{\$20\ per\ MWh}{(0.5,0.3)}, \dfrac{140 - 400\ gCO_2}{(0.4,0.5)}, \dfrac{100BTC}{(0.2,0.7)} >, \\ \mathcal{R}^4 < \dfrac{\$20\ per\ MWh}{(0.41,0.4)}, \dfrac{140 - 400\ gCO_2}{(0.7,0.3)}, \dfrac{100BTC}{(0.6,0.3)} >, \\ \mathcal{R}^5 < \dfrac{\$20\ per\ MWh}{(0.2,0.5)}, \dfrac{140 - 400\ gCO_2}{(0.3,0.6)}, \dfrac{100BTC}{(0.9,0.4)} > \end{array}\right\}.$$

$\mathcal{D}^2$ defined q − ROFHS values to the parametric choices as:

$$\mathcal{W}^2 = \left\{\begin{array}{l} \mathcal{R}^1 < \dfrac{\$20\ per\ MWh}{(0.2,02)}, \dfrac{140 - 400\ gCO_2}{(0.5,0.4)}, \dfrac{100BTC}{(0.5,0.7)} >, \\ \mathcal{R}^2 < \dfrac{\$20\ per\ MWh}{(0.4,0.7)}, \dfrac{140 - 400\ gCO_2}{(0.9,0.7)}, \dfrac{100BTC}{(0.8,0.9)} >, \\ \mathcal{R}^3 < \dfrac{\$20\ per\ MWh}{(0.5,0.3)}, \dfrac{140 - 400\ gCO_2}{(0.7,0.7)}, \dfrac{100BTC}{(0.5,0.8)} >, \\ \mathcal{R}^4 < \dfrac{\$20\ per\ MWh}{(0.1,0.3)}, \dfrac{140 - 400\ gCO_2}{(0.6,0.3)}, \dfrac{100BTC}{(02,0.1)} >, \\ \mathcal{R}^5 < \dfrac{\$20\ per\ MWh}{(0.2,0.5)}, \dfrac{140 - 400\ gCO_2}{(0.4,0.4)}, \dfrac{100BTC}{(0.5,0.5)} > \end{array}\right\}.$$

Step 3: The DM choice and expertise-based weights for each attribute were $\omega_j = (0.229, 0.471, 0.300)$.

Step 4: The weighted sum $\mathcal{S}_j = \sum_{j=1}^{k} \mathcal{N}_{ij}^{l}$ for each criterion ($\mathcal{C}_i$) was given in Table 2.

Table 2
The resulting weighted sums

Alternatives	$\mathcal{S}_1$	$\mathcal{S}_2$
$\mathcal{R}^1$	0.323	0.732
$\mathcal{R}^2$	0.452	0.352
$\mathcal{R}^3$	0.521	0.625
$\mathcal{R}^4$	0.627	0.928
$\mathcal{R}^5$	0.234	0.736

Step 5: Finally, list the alternatives with total scores $\max(\mathcal{S}_i)$ and rank the highest value. $Score = \{\mathcal{R}^1 < 0.732 >, \mathcal{R}^2 < 0.452 >, \mathcal{R}^3 < 0.625 >, \mathcal{R}^4 < 0.928 >, \mathcal{R}^5 < 0.736 >\}$. The ranking findings show that when using renewable energy sources to fuel Bitcoin mining, the order of efficacy is $\mathcal{R}^2 < \mathcal{R}^3 < \mathcal{R}^1 < \mathcal{R}^5 < \mathcal{R}^4$. This strategy could have two beneficial effects: (*i*) it could significantly lower carbon footprints, and (*ii*) boost economic growth. The energy-intensive nature of Bitcoin mining can be addressed by using sustainable energy sources like solar, wind, or hydroelectric power in processing processes. This change could significantly lessen the negative effects that these activities have on the environment.

Utilizing renewable energy for Bitcoin mining promotes employment and innovation in the renewable energy sector in addition to making the environment greener. This is in line with larger international efforts to tackle climate change and create a more environmentally sustainable future. The ranking results are shown in Figure 3, which provides visual evidence of the possible advantages of using renewable energy in Bitcoin mining operations.

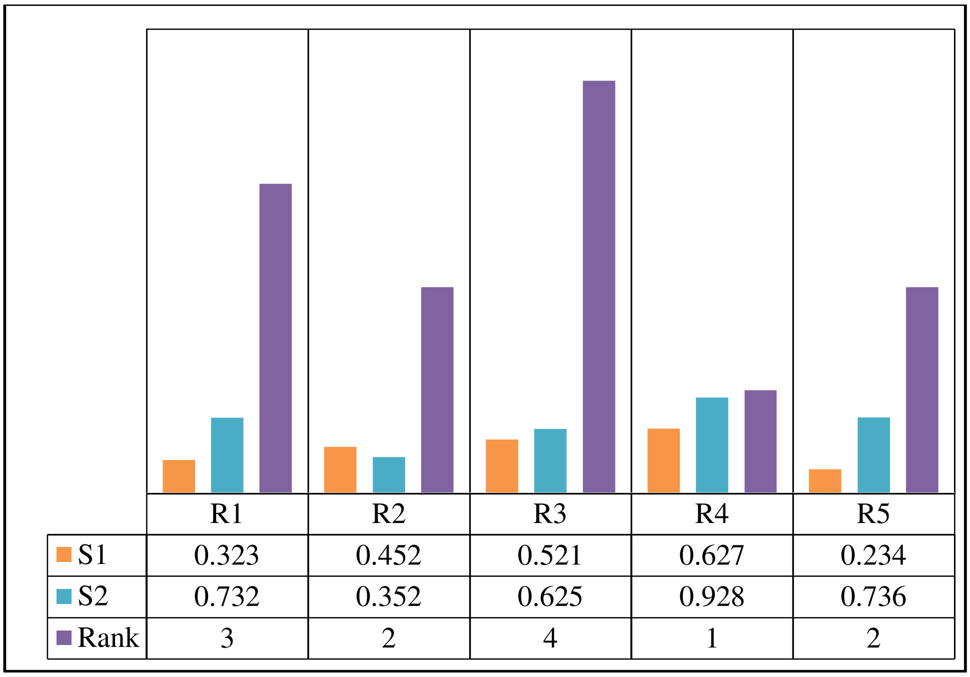

Fig. 3. Renewable energy alternative rankings

5. Discussion and Comparison

In this study, we proposed the q-ROFHS-based AHP algorithm to solve MCDM problems. The formulated algorithm is more useful for prediction and implementation. The study focuses on the divided attributes, emphasizing how flexible it is to changing choice factors, attributes, and outputs for decision-makers. It emphasizes how crucial it is to rank techniques across various models so that they can be directly compared based on predictions. The study presents the result of the comparison with q-ROF hybrid set structures. It also highlights how incomplete and unclear facts are frequently included in decision-making processes, highlighting the necessity for a technique to communicate information more precisely and logically.

To demonstrate the utility of the premeditated technique, we equate the achieved significance with some dominant methods under the setting q-ROFS, q-ROFSS, and q-ROFHS (Table 3). According to Table 3, the suggested approach is expected to outperform several q-ROF hybrid set structures in terms of effectiveness, importance, superiority, and improvement.

Table 3
Comparison of the proposed q-ROFHS-based AHP algorithm

Set	*MCDM algorithm*	*Parametrization*	*Sub-attributes*
q-ROFS [24]	**AHP**	×	×
q-ROFSS [28]	**AHP**	✓	×
q-ROFHS [*proposed*]	**AHP**	✓	✓

The efficacy of methods for mitigating pollution is contingent upon optimization algorithms that include variables like economic viability and the incorporation of sustainable energy sources. Decision support systems that use AHP provide stakeholders with data-driven insights. The passage is in line with the problems that cryptocurrency mining poses for the environment in this context, especially because of the heavy reliance on fossil fuels and non-renewable energy sources that result in large carbon footprints. The suggested method highlights the special qualities of q-ROFHS and associated AOs as essential instruments for decision-making in reducing the carbon footprint issues related to Bitcoin mining to meet this urgency.

6. Conclusion

The incorporation of MCDM techniques is essential for tackling the significant environmental problems associated with carbon emissions from cryptocurrency mining. MCDM provides a proactive approach to analyzing large information, finding trends, and forecasting future emissions. The most successful pollution reduction techniques are determined using optimization algorithms, which consider elements like cost-effectiveness and the integration of renewable energy. Decision support systems give stakeholders data-driven insights by using AHP. MCDM is useful for feasibility studies, scenario simulations, and ongoing monitoring since it guarantees the applicability and flexibility of suggested tactics. Because of the use of fossil fuels and non-renewable energy sources, this activity has a substantial environmental impact and increases carbon footprints.

Considering the urgency of the problem, our work introduced the q-ROFHS-based AHP algorithm as a potential solution. We implemented the q-ROFHS AOs in a real-world context, going beyond theoretical advances. The evaluation of carbon footprints associated with cryptocurrency mining was solved with q-ROFHS-based AHP. This method assured a thorough assessment that took cost-effectiveness and practicality into account, in addition to identifying and prioritizing the best pollution reduction techniques. Our suggested method attempted to lessen the negative effects of

Bitcoin mining on the environment by combining energy-saving technologies with renewable energy sources. Our study emphasized the requirement of implementing eco-friendly methods in the changing landscape of cryptocurrency mining. It provided a crucial contribution towards discovering practical and significant solutions as we traversed the issues faced by carbon emissions.

References

[1] Anandhabalaji, V., Babu, M., & Brintha, R. (2024). Energy Consumption by Cryptocurrency: A Bibliometric Analysis Revealing Research Trends and Insights. *Energy Nexus, 13*, 100274. https://doi.org/10.1016/j.nexus.2024.100274.

[2] Krause, M., Tolaymat, T., & Katz, G. (2020). The Carbon Footprint of Bitcoin. *Joule, 4*(5), 1177-1186. https://doi.org/10.1016/j.joule.2019.05.012.

[3] Zadeh, L. A. (1965). Fuzzy Sets. *Information and Control, 8*, 338-353. https://doi.org/10.1016/S0019-9958(65)90241-X.

[4] Atanassov, K. (1986). Intuitionistic Fuzzy Sets. *Fuzzy Sets and Systems, 20*, 87–96.

[5] Wang, W., & Liu, X. (2011). Intuitionistic fuzzy geometric aggregation operators based on Einstein operations. *International Journal of Intelligent Systems, 26*(11), 1049-1075. https://doi.org/10.1002/int.20498.

[6] Xu, Z.S. (2007). Intuitionistic fuzzy aggregation operators. *IEEE Transactions on Fuzzy Systems, 5*, 1179-1187. https://doi.org/10.1109/TFUZZ.2006.890678.

[7] De, S.K., Biswas, R., & Roy, A.R. (2000). Some operations on intuitionistic fuzzy sets. *Fuzzy Sets and Systems, 114*(3), 477-484. https://doi.org/10.1016/S0165-0114(98)00191-2.

[8] Lin, L., Yuan, X. H., & Xia, Z. Q. (2007). Multicriteria fuzzy decision-making methods based on intuitionistic fuzzy sets. *Journal of Computer and System Sciences, 73*(1), 84-88. https://doi.org/10.1016/j.jcss.2006.03.004.

[9] Garg, H., (2018). An improved cosine similarity measure for intuitionistic fuzzy sets and their applications to decision-making process. *Hacettepe Journal of Mathematics and Statistics, 47*(6), 1578-1594. https://doi.org/10.15672/HJMS.2017.510.

[10] Yager, R. R. (2016). Generalized orthopair fuzzy sets. *IEEE Transactions on Fuzzy Systems, 25*(5), 1222-1230. https://doi.org/10.1109/TFUZZ.2016.2604005.

[11] Molodtsov, D. (1999). Soft set theory—first results. *Computers & Mathematics with Applications, 37*(4-5), 19-31. https://doi.org/10.1016/S0898-1221(99)00056-5.

[12] Maji, P. K., Biswas, R., & Roy, A.R. (2003). Soft set theory. *Computers & Mathematics with Applications, 45*(4-5), 555-562. https://doi.org/10.1016/S0898-1221(03)00016-6.

[13] Cagman, N., & Enginoglu, S. (2011). FP-soft set theory and its applications. *Annals of Fuzzy Mathematics and Informatics, 2*(2), 219-226.

[14] Ali, M.I., Feng, F., Liu, X., Min, W.K., & Shabir, M. (2009). On some new operations in soft set theory. *Computers & Mathematics with Applications, 57*(9), 1547-1553. https://doi.org/10.1016/j.camwa.2008.11.009.

[15] Maji, P.K., Biswas, R., & Roy, A.R. (2001). Fuzzy soft sets. *Journal of Fuzzy Mathematics, 9*, 589–602.

[16] Roy, A.R., & Maji, P.K. (2007). A fuzzy soft set theoretic approach to decision making problems. *Journal of Computational and Applied Mathematics, 203*(2), 412-418. https://doi.org/10.1016/j.cam.2006.04.008.

[17] Cagman, N., Enginoglu, S., & Citak, F. (2011). Fuzzy soft set theory and its applications. *Iranian Journal of Fuzzy Systems, 8*(3), 137-147.

[18] Feng, F., Jun, Y. B., Liu, X., & Li, L. (2010). An adjustable approach to fuzzy soft set-based decision making. *Journal of Computational and Applied Mathematics, 234*(1), 10-20. https://doi.org/10.1016/j.cam.2009.11.055.

[19] Maji, P.K., Biswas, R., & Roy, A.R. (2001). Intuitionistic fuzzy soft sets. *Journal of Fuzzy Mathematics, 9*, 677–692.

[20] Çağman, N., & Karataş, S. (2013). Intuitionistic fuzzy soft set theory and its decision making. *Journal of Intelligent & Fuzzy Systems, 24*(4), 829-836.

[21] Arora, R., & Garg, H. (2018). A robust aggregation operator for multi-criteria decision-making with intuitionistic fuzzy soft set environment. *Scientia Iranica, 25*(2), 931-942. https://doi.org/10.24200/sci.2017.4433.

[22] Muthukumar, P., & Krishnan, G.S.S. (2016). A similarity measure of intuitionistic fuzzy soft sets and its application in medical diagnosis. *Applied Soft Computing, 41*, 148-156. https://doi.org/10.1016/j.asoc.2015.12.002.

[23] Yager, R. R., Alajlan, N., & Bazi, Y. (2018). Aspects of generalized orthopair fuzzy sets. *International Journal of Intelligent Systems, 33*(11), 2154-2174. https://doi.org/10.1002/int.22008.

[24] Ali, M.I. (2018). Another view on q-rung orthopair fuzzy sets. *International Journal of Intelligent Systems, 33*(11), 2139-2153. https://doi.org/10.1002/int.22007.

[25] Liu, P., & Wang, P. (2018). Some q-rung orthopair fuzzy aggregation operators and their applications to multiple-attribute decision making. *International Journal of Intelligent Systems, 33*(2), 259-280. https://doi.org/10.1002/int.21927.

[26] Bede, B. (2013). General q-rung orthopair fuzzy linguistic variables and their application in decision-making. *Knowledge-Based Systems, 37*, 283-294.

[27] Bede, B., & Šipošová, A. (2020). Multi-criteria decision-making using q-rung orthopair fuzzy linguistic variables. *Expert Systems with Applications, 146*, 113208.

[28] Hussain, A., Ali, M. I., Mahmood, T., & Munir, M. (2020). q-Rung orthopair fuzzy soft average aggregation operators and their application in multi-criteria decision-making. *International Journal of Intelligent Systems, 35*(4), 571-599. https://doi.org/10.1002/int.22217.

[29] Smarandache, F. (2018). Extension of soft set to hypersoft set, and then to plithogenic hypersoft set. *Neutrosophic Sets and Systems, 22*, 168-170.

[30] Saqlain M, Sana M, Jafar N, Saeed. M, Said. B, (2020). Single and Multi-valued Neutrosophic Hypersoft set and Tangent Similarity Measure of Single valued Neutrosophic Hypersoft Sets. *Neutrosophic Sets and Systems, 32*, 317-329. https://doi.org/10.5281/zenodo.3723165.

[31] Saqlain, M., Riaz, M., Kiran, N., Kumam, P., & Yang, M.S. (2023). Water Quality Evaluation Using Generalized Correlation Coefficient for M-Polar Neutrosophic Hypersoft Sets. *Neutrosophic Sets and Systems, 55*, 58-89. https://doi.org/10.5281/zenodo.7832716.

[32] Riaz, M., Habib, A., Saqlain, M., & Yang, M.S. (2023). Cubic bipolar fuzzy-VIKOR method using new distance and entropy measures and Einstein averaging aggregation operators with application to renewable energy. *International Journal of Fuzzy Systems, 25*(2), 510-543. https://doi.org/10.1007/s40815-022-01383-z.

[33] Saqlain, M., Riaz, M., Imran, R., & Jarad, F. (2023). Distance and similarity measures of intuitionistic fuzzy hypersoft sets with application: Evaluation of air pollution in cities based on air quality index. *AIMS Mathematics, 8*(3), 6880-6899.

[34] Saqlain, M., Garg, H., Kumam, P., & Kumam, W. (2023). Uncertainty and decision-making with multi-polar interval-valued neutrosophic hypersoft set: A distance, similarity measure, and machine learning approach. *Alexandria Engineering Journal, 84*, 323-332. https://doi.org/10.1016/j.aej.2023.11.001.

[35] Saqlain, M., Kumam, P., Kumam, W., & Phiangsungnoen, S. (2023). Proportional Distribution Based Pythagorean Fuzzy Fairly Aggregation Operators with Multi-Criteria Decision-Making. *IEEE Access, 11*, 72209-72226. https://doi.org/10.1109/ACCESS.2023.3292273.

[36] Saqlain, M., Kumam, P., & Kumam, W. (2023). Linguistic Hypersoft Set with Application to Multi-Criteria Decision-Making to Enhance Rural Health Services. *Neutrosophic Sets and Systems, 61*, 28-52. https://doi.org/10.5281/zenodo.10428591.

[37] Khan, S., Gulistan, M., & Wahab, H. A. (2021). Development of the structure of q-Rung Orthopair Fuzzy Hypersoft Set with basic Operations. *Punjab University Journal of Mathematics, 53*(12), 881-892. https://doi.org/10.52280/pujm.2021.531204.

[38] Zulqarnain, R.M., Rehman, H.K.U., Awrejcewicz, J., Ali, R., Siddique, I., Jarad, F., & Iampan, A. (2022). Extension of Einstein average aggregation operators to medical diagnostic approach under Q-rung orthopair fuzzy soft set. *IEEE Access, 10*, 87923-87949. https://doi.org/10.1109/ACCESS.2022.3199069.

[39] Zulqarnain, R.M., Siddique, I., Eldin, S.M., & Gurmani, S.H. (2022). Extension of interaction aggregation operators for the analysis of cryptocurrency market under q-rung orthopair fuzzy hypersoft set. *IEEE Access, 10*, 126627-126650. https://doi.org/10.1109/ACCESS.2022.3224050.

[40] Gurmani, S.H., Chen, H., & Bai, Y. (2023). Extension of TOPSIS Method Under q-Rung Orthopair Fuzzy Hypersoft Environment Based on Correlation Coefficients and Its Applications to Multi-Attribute Group Decision-Making. *International Journal of Fuzzy Systems, 25*, 1-14. https://doi.org/10.1007/s40815-022-01386-w.

[41] Zulqarnain, R.M., Siddique, I., Mahboob, A., Ahmad, H., Askar, S., & Gurmani, S.H. (2023). Optimizing construction company selection using einstein weighted aggregation operators for q-rung orthopair fuzzy hypersoft set. *Scientific Reports, 13*(1), 1-43. https://doi.org/10.1038/s41598-023-32818-8.

[42] Ying, C., Slamu, W., & Ying, C. (2022). Multi-Attribute Decision Making with Einstein Aggregation Operators in Complex Q-Rung Orthopair Fuzzy Hypersoft Environments. *Entropy, 24*(10), 1494. https://doi.org/10.3390/e24101494.