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# An Alternative Approach for Enhanced Decision-Making using Fermatean Fuzzy Sets

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**ABSTRACT**

This paper explores an innovative approach to solving multi-criteria decision-making (MCDM) problems by focusing on the aggregation of membership and non-membership values using score functions of Fermatean fuzzy sets. Fermatean fuzzy sets have been used to provide a more complex and flexible framework for decision-making because of their ability to handle higher levels of uncertainty and ambiguity. The suggested approach makes use of the unique characteristics of Fermatean fuzzy sets to enhance the aggregation procedure and guarantee a more accurate representation of uncertainty in scenarios involving decision-making. The study begins with a comprehensive review of existing fuzzy set theories and their applications in MCDM, highlighting the limitations of traditional methods. The novel part of this work is to provide a score function designed for Fermatean fuzzy sets that improves the accuracy and consistency of the aggregation procedure. Such score function offers a reliable method for combining several criteria since they are carefully put together to represent the varied relationships between membership and non-membership values. To validate the effectiveness of the proposed approach, the methodology is applied to a hypothetical case study in software design. The results underscore the potential of Fermatean fuzzy sets in addressing the complexities of MCDM, particularly in fields requiring high levels of precision and adaptability. This research presents a novel and effective alternative to traditional MCDM approaches, offering significant improvements in the handling of uncertainty and ambiguity. Experimental results of the study are presented and compared with the existing literature.

## 1. Introduction

In the contemporary decision-making landscape, multi-criteria decision-making (MCDM) plays a pivotal role in addressing complex problems characterized by multiple conflicting criteria. MCDM has been extensively explored and utilized across various domains, including engineering, economics, and management, due to its capability to provide systematic and rational solutions [1]. However, the

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inherent uncertainty and imprecision in decision-making processes necessitate the use of advanced mathematical tools, such as fuzzy sets, to handle such ambiguities effectively. The concept of fuzzy sets, introduced by Zadeh [2], provided a framework for dealing with uncertainties by allowing partial membership of elements in sets. This foundational idea has been expanded into various forms, including intuitionistic fuzzy sets (IFS) and Pythagorean fuzzy sets (PFS), to better capture the degrees of membership and non-membership simultaneously. Atanassov's intuitionistic fuzzy sets [3-4] extend the classical fuzzy set theory by incorporating an additional degree of hesitation, thus offering a more comprehensive approach to modeling uncertainty. The advent of PFSs, proposed by Yager [5-6], introduced a novel dimension to fuzzy set theory by allowing the sum of the squares of the membership and non-membership degrees to be less than or equal to one. This extension enhances the flexibility and applicability of fuzzy sets in capturing uncertainty. PFS has been utilized in various MCDM methods to improve decision-making accuracy and robustness [7-8].

The concept of Fermatean fuzzy sets has been developed as an extension of the existing fuzzy set theories to address more complex and uncertain information in decision-making processes. Initiated by Senapati and Yager [9-11], these sets were conceptualized to provide a more flexible framework than traditional fuzzy and intuitionistic fuzzy sets. Fermatean fuzzy sets have been characterized by a membership degree, a non-membership degree, and a hesitancy degree, offering a richer structure for modeling uncertainty. The formulation of these sets has been based on the generalization of PFSs, which themselves extended IFSs. The enhancement provided by Fermatean fuzzy sets lies in their ability to handle more intricate uncertainties without compromising computational efficiency or the robustness of the decision-making process [11]. Several operations over Fermatean fuzzy numbers have been explored to enrich their applicability. Senapati and Yager [9] introduced new operations on Fermatean fuzzy numbers, expanding the algebraic structures that can be utilized in various computational scenarios. These operations were designed to enhance the manipulation of Fermatean fuzzy data, thus improving the accuracy and reliability of outcomes in MCDM processes. The operations have included, but are not limited to, addition, multiplication, and scalar multiplication, each tailored to preserve the inherent properties of Fermatean fuzzy numbers while ensuring their practical applicability in decision-making models [10].

The domain of MCDM has been extensively explored through various techniques aimed at optimizing decision processes in complex environments. These techniques have been developed to address the intricate nature of decision-making where multiple criteria must be considered and balanced. The literature presents a diverse array of approaches that have been utilized across different fields, each contributing to the evolution of MCDM methodologies. Hybrid fuzzy MCDM approaches have been a significant area of research, particularly in project selection problems. Salehi [12] introduced a hybrid fuzzy MCDM approach, combining the strengths of different methodologies to enhance decision accuracy. This approach integrated fuzzy logic to handle uncertainty and imprecision, making it suitable for real-world project selection scenarios where precise information is often unavailable. This methodology underscored the importance of hybrid approaches in addressing the multifaceted nature of decision-making problems. The introduction of fuzzy parameters in project selection has further enriched MCDM techniques. Huang [13] explored optimal project selection with random fuzzy parameters, highlighting the importance of incorporating both randomness and fuzziness in decision models. This approach has been crucial in environments where project parameters are uncertain and variable, offering a more realistic and adaptable decision-making process.

In the field of research and development (R&D), fuzzy approaches have been pivotal. Carlsson et al. [14] proposed a fuzzy approach to R&D project portfolio selection, emphasizing the need to handle the inherent uncertainty in R&D environments. This methodology has facilitated the evaluation and selection of R&D projects by incorporating fuzzy logic to represent imprecise information, thus improving the decision quality. The fuzzy analytic network process (ANP) has been another significant advancement in MCDM. Mohanty et al. [15] introduced a fuzzy ANP-based approach for R&D project selection, showcasing its capability to handle complex interdependencies among criteria. This approach has provided a structured framework for evaluating R&D projects, considering both qualitative and quantitative factors in the decision-making process.

A constrained fuzzy analytic hierarchy process (AHP) has been employed to refine project selection methodologies. Enea and Piazza [16] proposed a constrained fuzzy AHP model that enhanced project evaluation by incorporating constraints into the decision framework. This model has been crucial in ensuring that selected projects not only meet multiple criteria but also adhere to specific constraints, thus aligning with organizational policies and limitations. The integration of fuzzy AHP and TOPSIS techniques has further advanced project selection methodologies. Mahmoodzadeh et al. [17] utilized a combination of fuzzy AHP and TOPSIS to improve project evaluation processes. This hybrid approach has benefited from the strengths of both techniques, offering a robust and comprehensive framework for project selection that accommodates both fuzziness and ranking capabilities. Applications of AHP and fuzzy TOPSIS methods in specific industries were explored by Amiri [18], who applied these techniques to the oil-field development project selection. This study highlighted the practical applicability of MCDM techniques in industry-specific contexts, demonstrating their effectiveness in addressing complex decision-making problems in the oil and gas sector. The use of the fuzzy ELECTRE method in project selection has also been investigated. Daneshvar Rouyendegh and Erol [19] introduced this method to enhance project selection processes by considering multiple criteria and their respective weights. The fuzzy ELECTRE method has provided a robust decision-making tool that ranks projects based on their overall performance across various criteria.

Comparative analyses of different MCDM methods have been conducted to evaluate their effectiveness. Chu et al. [20] compared three analytical methods for knowledge communities' group decision analysis, highlighting the strengths and weaknesses of each method. Such comparative studies have been valuable in identifying the most suitable MCDM techniques for specific decision-making contexts. Compromise solutions in MCDM have been explored through methods like VIKOR and TOPSIS. Opricovic and Tzeng [21] conducted a comparative analysis of these methods, emphasizing their applicability in achieving compromise solutions. These methods have been instrumental in situations where decision-makers seek a balanced solution that satisfies multiple criteria to an acceptable degree. Portfolio optimization using hybrid MCDM methods has been another significant area of research. Raei and Jahromi [22] explored a hybrid approach combining fuzzy ANP, VIKOR, and TOPSIS for portfolio optimization. This approach has leveraged the strengths of multiple MCDM techniques to optimize project portfolios, ensuring that the selected projects align with strategic objectives and provide the best possible outcomes.

The development of Fermatean fuzzy sets has further enriched MCDM methodologies. Senapati and Yager [10] introduced Fermatean fuzzy weighted averaging and geometric operators, demonstrating their application in MCDM methods. These operators have enhanced the ability to aggregate criteria and make more informed decisions, particularly in scenarios involving higher degrees of uncertainty and complexity. Sahoo et al. [23] examined score function-based effective ranking of interval-valued Fermatean fuzzy sets, showcasing their applicability in multi-criteria decision-making problems.

The evolution of aggregation operators has been significantly influenced by the development of fuzzy set theory and its extensions, such as IFSs and PFSs. Several types of aggregation operators have been developed to handle fuzzy, intuitionistic fuzzy, Pythagorean fuzzy, and Fermatean fuzzy data.

These include the ordered weighted averaging (OWA) operator [24], the Choquet integral [25], and the Dombi aggregation operator [26]. Each of these operators has unique properties and is suited for different decision-making scenarios. For instance, the OWA operator, which provides a parameterized family of aggregation operators, allows decision-makers to model different types of aggregation behavior by adjusting the weights assigned to the inputs [24]. The Choquet integral, on the other hand, is particularly useful in scenarios where interactions among criteria need to be considered, as it allows for the aggregation of inputs for a fuzzy measure [25].

Recent research has focused on enhancing the capabilities of these traditional aggregation operators and developing new ones to address the growing complexity of decision-making problems. For example, Garg [27] introduced generalized Pythagorean fuzzy information aggregation operators using Einstein operations, which were effective in MCDM scenarios. Similarly, Liu [28] proposed Hamacher aggregation operators based on interval-valued intuitionistic fuzzy numbers, which provide a flexible approach to handling uncertainty in group decision-making processes. Furthermore, the concept of deviation-based aggregation functions, as explored by Decky et al. [29], offered an innovative approach to aggregation by considering the deviations of inputs from a central value. This approach was particularly useful in scenarios where maintaining a balance among criteria is crucial. Additionally, the introduction of super migrative aggregation functions by Durante and Ricci [30] opened new avenues for research in the field of aggregation operators, offering a framework that ensures the super additivity property, which is desirable in many decision-making contexts.

The application of these advanced aggregation operators has been demonstrated in various practical decision-making scenarios. For instance, Dong et al. [31] discussed the use of consensus-reaching processes in social network group decision-making, highlighting the challenges and research paradigms associated with this approach. The development of Fermatean fuzzy weighted averaging (FFWA) and Fermatean fuzzy weighted geometric (FFWG) operators marked a significant milestone in the application of these sets. These operators, as introduced by Senapati and Yager [9-11], have been pivotal in refining MCDM methods. By using the unique attributes of Fermatean fuzzy sets, these operators have facilitated more accurate aggregation of criteria, leading to more robust and reliable decision outcomes. The FFWA and FFWG operators have been applied in various engineering and artificial intelligence contexts, showcasing their versatility and effectiveness in dealing with complex decision-making scenarios [9-11].

This research aims to develop a novel MCDM approach that integrates Fermatean fuzzy sets with alternative aggregation techniques to enhance decision-making under uncertainty. In this paper, we present a novel aggregation approach called the membership and non-membership aggregation method. This method involves calculating the weighted average of the cubic power of membership and non-membership functions. Subsequently, we determine the score function value for each alternative using appropriate score functions. To compute the weights, we employ the entropy method as well as the score functions.

### *1.1 Background and Motivation*

Software design problems typically involve a multitude of criteria, each reflecting different aspects of the system's functionality and performance. Commonly considered criteria in software design include usability, performance, security, scalability, and maintainability. Each of these criteria plays a pivotal role in determining the overall quality and success of the software product. For instance, usability ensures that the end-users can efficiently and effectively interact with the system, while performance relates to the system's responsiveness and processing capabilities. Security addresses the system's ability to protect against unauthorized access and cyber threats, scalability pertains to the system's capability to handle increased loads, and maintainability reflects the ease with which the system can be modified and upgraded. The challenge in MCDM for software design lies in the subjective and often ambiguous nature of these criteria. Designers and stakeholders may have different perceptions and preferences, leading to varied evaluations of the same design alternative. Fuzzy set theory, provides a mathematical framework to model such uncertainties by allowing both membership and non-membership degrees, thereby offering a richer and more flexible representation of uncertainty. This study contributes to the field by offering a comprehensive approach to MCDM that integrates the strengths of Fermatean fuzzy sets and aggregation techniques, providing a robust solution to decision-making problems under uncertainty.

### *1.2 Research Objectives and Contributions*

This research aims to address the MCDM problem in software design by employing the Sahoo score function [32] to evaluate and rank four alternative designs based on five critical criteria; i.e. usability, performance, security, scalability, and maintainability. The primary contributions of this study are as follows:

i. *Development of a novel aggregation framework* – We propose a new aggregation framework that integrates the Sahoo score function [32] with simple Fermatean fuzzy aggregation approaches. This framework is designed to capture the complex interactions between membership and non-membership degrees, thereby providing a more accurate and reliable assessment of software design alternatives.
ii. *Application to software design* – We demonstrate the practical applicability of the proposed framework by solving a real-world MCDM problem in software design.
iii. *Validation and comparative analysis* – We validate the effectiveness of the proposed framework through a comparative analysis with existing methods.

The remainder of this paper is organized as follows: Section 2 introduces fundamental mathematical terms. Section 3 outlines the calculation of criteria weights using the entropy method. Section 4 illustrates the method for utilizing membership and non-membership functions aggregation in MCDM. Section 5 discusses various technical terms related to software design. Section 6 contains the numerical study, results, and discussions. Finally, Section 7 presents the conclusion.

## 2. Preliminaries

In this section, we define several fundamental mathematical terms that are used throughout the entire manuscript.

**Definition 2.1** [9-10]. Let $U$ be a universe of discourse. A Fermatean fuzzy set (FFS) $F$ in $U$ is given by $F=\{<u,\mu_F(u),\gamma_F(u)>: u\in U\}$, where $\mu_F : U \to [0,1]$ denotes the degree of membership and $\gamma_F : U \to [0,1]$ denotes the degree of non-membership of the element $u\in U$ to the set $F$, respectively. The degree of hesitation that the element $u\in U$ belongs to the FFS $F$ is denoted by $\pi_F(u)$ and is defined by $\pi_F(u)=\sqrt[3]{1-\mu_F^3(u)-\gamma_F^3(u)}$. Now, according to Senapati and Yager [9-10], we may denote $F=(\mu_F,\gamma_F)$ as Fermatean fuzzy number (FFN). Let $F=(\mu_F,\gamma_F)$ be the FFN, then hesitation is $\pi_F=\sqrt[3]{1-\mu_F^3-\gamma_F^3}$.

**Definition 2.2** [9-10]. Let $F=(\mu_F,\gamma_F)$, $F_1=(\mu_{F_1},\gamma_{F_1})$, and $F_2=(\mu_{F_2},\gamma_{F_2})$ are three PFNs and a scalar $\lambda>0$, then:

1. $F_1+F_2=(\sqrt[3]{\mu_{F_1}^3+\mu_{F_2}^3-\mu_{F_1}^3\mu_{F_2}^3},\gamma_{F_1}\gamma_{F_2})$,
2. $F_1F_2=(\mu_{F_1}\mu_{F_2},\sqrt[3]{\gamma_{F_1}^3+\gamma_{F_2}^3-\gamma_{F_1}^3\gamma_{F_2}^3})$,
3. $\lambda F=(\sqrt[3]{1-(1-\mu_F^3)^\lambda},\gamma_P^\lambda)$,
4. $F^\lambda=(\mu_F^\lambda,\sqrt[3]{1-(1-\gamma_F^3)^\lambda})$,
5. $F_1\cup F_2=(\max\{\mu_{F_1},\mu_{F_2}\},\min\{\gamma_{F_1},\gamma_{F_2}\})$,
6. $F_1\cap F_2=(\min\{\mu_{F_1},\mu_{F_2}\},\max\{\gamma_{F_1},\gamma_{F_2}\})$,
7. $F^C=(\gamma_F,\mu_F)$.

**Definition 2.3** [9-10]. Let $F=(\mu_F,\gamma_F)$ be the FFN, then the accuracy degree can be defined as $H(F)=\mu_F^3+\gamma_F^3$. Here, it is to be mentioned that $H(F)\in[0,1]$.

**Definition 2.4** [9-10]. Let $F=(\mu_F,\gamma_F)$ be the FFN, then the score function can be defined as $S(F)=\mu_F^3-\gamma_F^3$. It is to be mentioned that $S(F)\in[-1,1]$. It is to be noted that $H(P)\in[0,1]$, whereas $S(F)\in[-1,1]$. Here, it is to be mentioned that a function $F(.)$ is positive if $F(.)\in[0,1]$ and negative if $F(.)\in[-1,0)$. In the purpose of ranking, most of the researchers have considered score function lies in the interval -1 and 1. Also, some of the researchers have proposed score function lies in the interval 0 and 1. For more details, one may refer to the work of Sahoo [33].

**Definition 2.5** [33]. Let $F=(\mu_F,\gamma_F)$ be the FFN, then $S(F)$ can be defined as follows:

$$S(F)=\frac{1}{2}(1+\mu_F^3-\gamma_F^3). \tag{1}$$

**Theorem 2.1.** The score function $S(F)=\frac{1}{2}(1+\mu_F^3-\gamma_F^3)$ is positive; i.e. $S(F)\in[0,1]$.

**Proof of Theorem 2.1:** For any FFN $F=(\mu_F,\gamma_F)$, $0\le\mu_F^3$, and $0\le{\gamma^3}_F\le1$. Consequently $\gamma_F^3\le1\Rightarrow0\le1-\gamma_F^3$ and obviously $1+\mu_F^3-\gamma_F^3\ge0$. Also, $\mu_F^3+\gamma_F^3\le1\Rightarrow1+\mu_F^3+\gamma_F^3\le2$ $\Rightarrow1+\mu_F^3-\gamma_F^3\le2$. So, we may write $0\le\frac{1+\mu_F^3-\gamma_F^3}{2}\le1$. Therefore, $0\le\frac{1}{2}(1+\mu_F^3-\gamma_F^3)\le1$ $\Rightarrow S(F)\in[0,1]$.

**Definition 2.6.** Let $F_1=(\mu_{F_1},\gamma_{F_1})$ and $F_2=(\mu_{F_2},\gamma_{F_2})$ be two FFNs. Then ranking or order relation between $F_1$ and $F_2$ are as follows:

1. $F_1\succ F_2$ **if and only if either** $S(F_1)>S(F_2)$ **or** $S(F_1)=S(F_2)$ **and** $H(F_1)>H(F_2)$.
2. $F_1\prec F_2$ **if and only if either** $S(F_1)<S(F_2)$ **or** $S(F_1)=S(F_2)$ **and** $H(F_1)<H(F_2)$.

3. $F_1 \approx F_2$ **iff** $S(F_1)=S(F_2)$ **and** $H(F_1)=H(F_2)$.

## 3. Weight Calculation for Criteria using the Entropy Method

This section outlines a step-by-step method for calculating criteria weights using an entropy-based approach. For more details, please refer to Shannon's work [34]. If $D=(S(P_{ij}))_{m\times n}$ be the decision matrix and $w=(w_1,w_2,...,w_n)^T$ is the weight such that $0\le w_j \le 1$ along with $\sum_{j=1}^{n} w_j =1$. Following are the different steps to calculate criteria weights $w_j, j=1,2,...,n$.

**Step 1:** For $j=1,2,...,n$ calculate $E_j=-\frac{1}{\log(m)}\sum_{i=1}^{m} p_{ij}\log(p_{ij})$, where $p_{ij}=\frac{S(F_{ij})}{\sum_{j=1}^{n} S(F_{ij})}$. It is worth mentioning that $\lim_{p_{ij}\to 0} p_{ij}\log p_{ij} \to 0$.

**Step 2:** For $j=1,2,...,n$ calculate $w_j=\frac{1-E_j}{\sum_{j=1}^{n} 1-E_j}$.

## 4. Method to use membership and non-membership functions aggregation in MCDM

This section describes how membership and non-membership functions aggregation can be applied to solve an MCDM problem when the criteria weights are unknown, with the weights determined using Shannon entropy. Let $D=(F_{ij})_{m\times n}$ be the decision matrix. Here, each element of the decision matrix be expressed in terms of FFNs as $F_{ij}=(\mu_{ij},\gamma_{ij})$. Now, using the entropy method we calculated the criteria weights $w=(w_1,w_2,...,w_m)^T$. After that, we employed membership and non-membership functions aggregation and derived score values of each alternative $A_i, i=1,2,...,m$. We first computed the weighted average of the membership and non-membership degrees for each alternative $A_i$ and then we computed the corresponding scoring function values. Based on this, the choice with the greatest score function value is the most effective selection.

**Step 1** – *Form the decision matrix*. Let there are $m$ alternatives $A_1,A_2,...,A_m$ and $n$ criteria $C_1,C_2,...,C_n$ and $F_{ij}=(\mu_{ij},\gamma_{ij})$ is either benefit or cost for the decision makers' point of view. Then, the decision matrix (DM) is $D=(F_{ij})_{m\times n}$.

**Step 2** – *Calculate the score function value of each* $F_{ij}$. Then, normalized DM can be written as $S((F_{ij})_{m\times n})$. Here, $S(P_{ij})=S((\mu_{ij},\gamma_{ij}))$ is the score function value, which is either benefit or cost in view of decision-makers under uncertainty.

**Step 3** – *Calculate the weight vector* $w=(w_1,w_2,...,w_n)^T$ *using the entropy method.*

**Step 4** – *Calculate the weighted average of the membership degrees to the power of three for each alternative* $A_i$. This can be done using the following formula:

$$\mu_{A_i}^3 = \frac{\sum_{j=1}^{n} w_j \mu_{ij}^3}{\sum_{j=}^{n} w_j} \tag{2}$$

**Step 5** – *Calculate the weighted average of the non-membership degrees to the power of three for each alternative* $A_i$. This can be done using the following formula:

$$\gamma_{A_i}^3 = \frac{\sum_{j=1}^{n} w_j \gamma_{ij}^3}{\sum_{j=}^{n} w_j} \tag{3}$$

**Step 6** – *Calculate* $\mu_{A_i}$ *and* $\gamma_{A_i}$ *from Steps 3-4.*

**Step 7 –** *Calculate* $S(A_i)$ *for each alternative* $A_i$ *using Definition 2.5.*

**Step 8** – *The best choice can be identified by ranking the alternatives based on their score values, with the alternative having the highest score being the most desirable.*

## 5. Several Technical Terms Related to Software Design

In this section, we discuss several terms that are useful for designing a software system. We have also interpreted these terms within the framework of Fermatean fuzzy set theory.

### *5.1 Usability*

Usability in software design refers to how easily users can interact with the software to accomplish their goals efficiently. It involves intuitive interface design, ease of learning, error prevention, and user satisfaction. High usability means users can quickly learn how to use the software, perform tasks with minimal effort, and recover from errors without frustration. Usability testing with real users helps identify issues and improve the design. In terms of fuzzy sets, usability can be represented with a membership function that quantifies the degree to which a software system meets usability criteria. For example, a usability score might range from 0 (not usable) to 1 (highly usable), capturing the nuances of user experience. A fuzzy value like (0.7, 0.2) indicates a 70% usability score, with a 20% non-usability score, reflecting the uncertainty or variability in user feedback.

### *5.2 Performance*

Performance in software design measures how well the software responds to user inputs and processes data efficiently. It includes aspects like load time, response time, and throughput. High-performance software provides quick feedback to user actions, handles multiple tasks simultaneously, and operates smoothly under varying conditions. Performance testing evaluates the software's behaviour under normal and peak loads to ensure it meets expected standards. In fuzzy set terms, performance can be expressed with a membership function indicating the degree of performance quality. For instance, a performance score could range from 0 (poor performance) to 1 (excellent performance). A fuzzy value like (0.8, 0.15) might denote an 80% performance score, with a 15% score for poor performance, reflecting the performance variability under different conditions.

### *5.3 Security*

Security in software design focuses on protecting the software and its data from unauthorized access and threats. It involves implementing measures such as encryption, authentication, and authorization to ensure data confidentiality, integrity, and availability. Secure software prevents data breaches, protects user privacy, and ensures that only authorized users can access sensitive information. Security assessments and regular updates help maintain robust protection against evolving threats. In fuzzy set terms, security can be represented by a membership function that quantifies the security level. For example, a security score might range from 0 (insecure) to 1 (highly secure). A fuzzy value like (0.9, 0.05) could indicate a 90% security score, with a 5% score for insecurity, reflecting uncertainties in the security assessment process.

### *5.4 Scalability*

Scalability in software design refers to the system's ability to handle increasing workloads and expand its resources to accommodate growth. Scalable software can efficiently manage more users, data, or transactions without performance degradation. This involves designing the system architecture to support horizontal or vertical scaling, load balancing, and distributed computing. Scalability ensures that the software remains responsive and reliable as demand grows. In fuzzy set terms, scalability can be represented by a membership function indicating the scalability level. For example, a scalability score might range from 0 (not scalable) to 1 (highly scalable). A fuzzy value like (0.75, 0.2) could denote a 75% scalability score, with a 20% score for non-scalability, reflecting the variability in the system's ability to scale under different conditions.

### *5.5 Maintainability*

Maintainability in software design refers to how easily the software can be updated, modified, and managed over time. High maintainability means developers can quickly fix bugs, add new features, and adapt the software to changing requirements. Factors contributing to maintainability include clean code, modular design, comprehensive documentation, and adherence to coding standards. Maintainable software reduces technical debt, improves development efficiency, and extends the software's lifespan. In fuzzy set terms, maintainability can be represented by a membership function indicating the ease of maintenance. For example, a maintainability score might range from 0 (difficult to maintain) to 1 (easy to maintain). A fuzzy value like (0.65, 0.30) could indicate a 65% maintainability score, with a 30% score for difficulty in maintenance, reflecting uncertainties in the maintenance process.

## 6. Numerical Example

In software engineering, selecting an optimal design from multiple alternatives requires careful consideration of various criteria. This case study explores four software design alternatives (A1, A2, A3, and A4) evaluated against five key criteria: Usability (C1), Performance (C2), Security (C3), Scalability (C4), and Maintainability (C5). The evaluation employs a Fermatean fuzzy decision matrix to handle uncertainty and imprecision in the assessment process.

Software design alternatives are:

i. A microservices-based architecture (A1).
ii. A monolithic architecture (A2).
iii. A serverless architecture (A3).
iv. A service-oriented architecture (A4).

Evaluation criteria are:

i. Usability (C1) – The ease of use and user experience of the software.
ii. Performance (C2) – The efficiency and speed of the software.
iii. Security (C3) – The ability of the software to protect against threats.
iv. Scalability (C4) – The capacity of the software to handle growth.
v. Maintainability (C5) – The ease with which the software can be maintained and updated.

***(a) Criteria definition*** – All evaluation criteria are benefits, such as usability (C1), performance (C2), security (C3), scalability (C4), and maintainability (C5). In this example, there were no cost criteria.

***(b) Fuzzy number representation*** – Each criterion's value is represented as a Fermatean Fuzzy Number $F=(\mu_F,\gamma_F)$, where $\mu_F$ is the degree of membership and $\gamma_F$ is the degree of non-membership of the Fermatean fuzzy set.

***(c) Scoring functions –*** They were applied as $S(F)=\frac{1}{2}(1+\mu_F^3-\gamma_F^3)$.

***(d) Solution procedure –*** Method to use membership and non-membership functions aggregation in MCDM was discussed in Section 4. For this problem, the Fermatean fuzzy decision matrix was provided in Table 1. Each criterion is evaluated on a scale from 0 to 1, where 0 represents the lowest and 1 the highest score.

**Table 1**
Fermatean fuzzy decision matrix

| Alternatives/criteria | Usability (C1) | Performance (C2) | Security (C3) | Scalability (C4) | Maintainability (C5) |
|---|---|---|---|---|---|
| Design A1 | (0.70, 0.20) | (0.60, 0.30) | (0.80, 0.15) | (0.75, 0.20) | (0.65, 0.25) |
| Design A2 | (0.65, 0.25) | (0.70, 0.20) | (0.75, 0.20) | (0.70, 0.25) | (0.60, 0.30) |
| Design A3 | (0.8, 0.15) | (0.75, 0.20) | (0.65, 0.25) | (0.80, 0.15) | (0.70, 0.20) |
| Design A4 | (0.75, 0.20) | (0.80, 0.15) | (0.70, 0.25) | (0.75, 0.20) | (0.65, 0.25) |

Using the entropy method mentioned here, we have calculated the weight vector as $w=(0.22,0.20,0.21,0.24,0.13)^T$ for the decision matrix provided in Table 2.

**Table 2**
Converted decision matrix based on score function

| Alternatives/criteria | Usability (C1) | Performance (C2) | Security (C3) | Scalability (C4) | Maintainability (C5) |
|---|---|---|---|---|---|
| Design A1 | 0.67 | 0.59 | 0.75 | 0.71 | 0.63 |
| Design A2 | 0.63 | 0.67 | 0.71 | 0.66 | 0.59 |
| Design A3 | 0.75 | 0.71 | 0.63 | 0.75 | 0.67 |
| Design A4 | 0.71 | 0.75 | 0.66 | 0.71 | 0.63 |

The, we calculated $(\mu_{A_i}, \gamma_{A_i})$ ($i$=1,…,4) using the method provided in Section 4. The detail computational results are provided in Table 3. From Table 3, it was observed that A3 and A4 are more like each other, as are A1 and A2.

**Table 3**
Computational results based on score function

| Alternatives | $\mu_{A_i}$ | $\gamma_{A_i}$ | Score value | Rank |
|---|---|---|---|---|
| **Design A1** | **0.71** | **0.23** | **0.676** | **3** |
| **Design A2** | **0.69** | **0.24** | **0.657** | **4** |
| **Design A3** | **0.75** | **0.20** | **0.707** | **1** |
| **Design A4** | **0.74** | **0.21** | **0.697** | **2** |

The clustering representation of the alternatives is depicted in Figure 1.

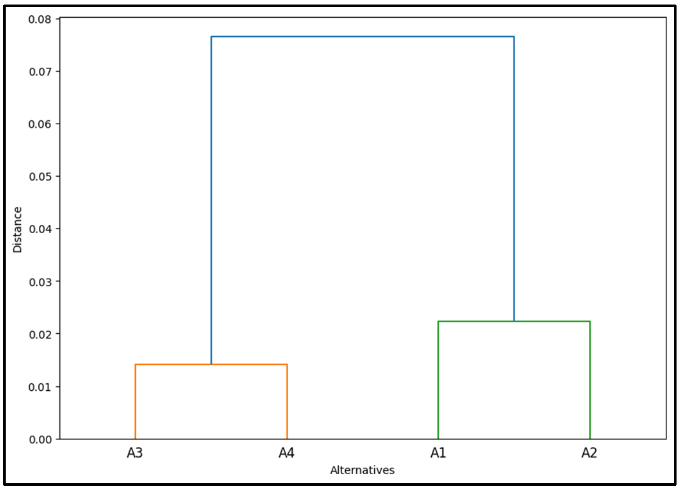


**Fig. 1.** Clustering representation of the alternatives

In Figure 2, the cluster centroid is marked (X), and the centroid coordinates are (0.7225, 0.22). In the horizontal direction, the points above the centroid are considered more suitable alternatives, whereas those below the centroid are given lesser priority.

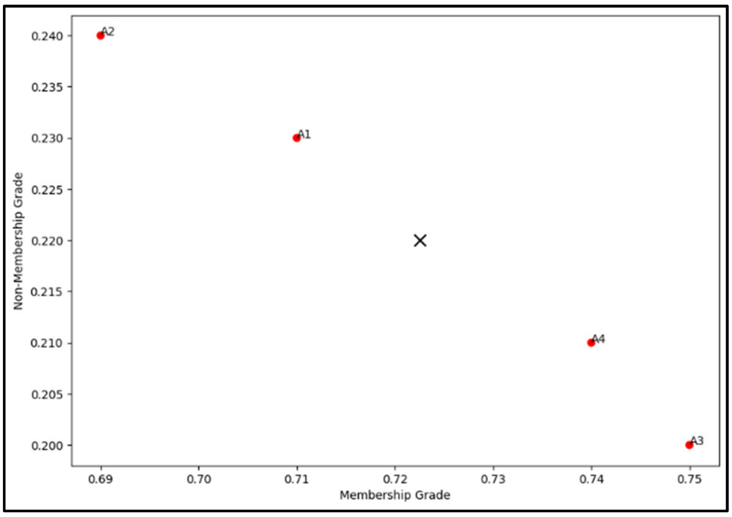


**Fig. 2.** Membership and non-membership grades and cluster centroid of the alternatives

The optimal design based on our score functions was achieved using our proposed approach. Detailed results are presented in Table 4. From Table 4, it is observed that our score function effectively determines the best design alternative.

**Table 4**
Results of the numerical experiment

| Used score function | Ranks of the alternatives | Best design | |
|---|---|---|---|
| **Type 1** | **Design A1** | $A_3 \succ A_4 \succ A_1 \succ A_2$ | $A_3$ |

A comparative study was conducted to enhance the reliability of our proposed approach. For this purpose, the numerical example studied by Senapati & Yager [10] was considered. This example was solved using our proposed approach. The obtained results are given in Table 5 as well as displayed in Figure 3. It was also noted that the same results were obtained using our proposed approach to solve the decision-making problems.

**Table 5**
Computational results based on type 1 score function

| Alternatives | $\mu_{A_i}$ | $\gamma_{A_i}$ | Score value | Rank |
|---|---|---|---|---|
| **S1** | **0.64** | **0.48** | **0.575** | **3** |
| **S2** | **0.68** | **0.61** | **0.549** | **4** |
| **S3** | **0.77** | **0.47** | **0.678** | **1** |
| **S4** | **0.67** | **0.46** | **0.604** | **2** |

From Figure 5, it was observed that S1 and S4 are more similar, and S3, S1, and A2 are also approximately similar.

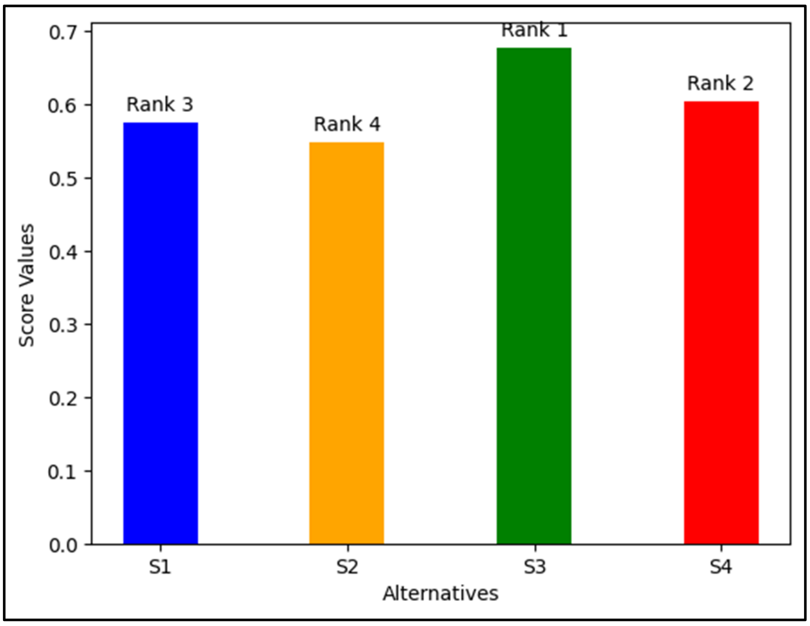


**Fig. 3.** Score values for different alternatives with ranks

The clustering representation of the alternatives S1, S2, S3, and S4 is depicted in Figure 4.

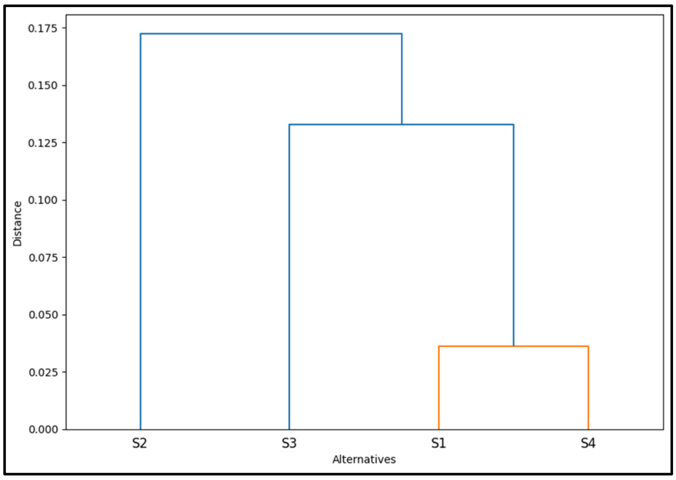


**Fig. 4.** Clustering representation of the alternatives S1 to S4

In Figure 5, the cluster centroid is marked (X), and the centroid coordinates are (0.69, 0.505). In the horizontal direction, the points above the centroid are considered more suitable alternatives, whereas those below the centroid are given lesser priority.

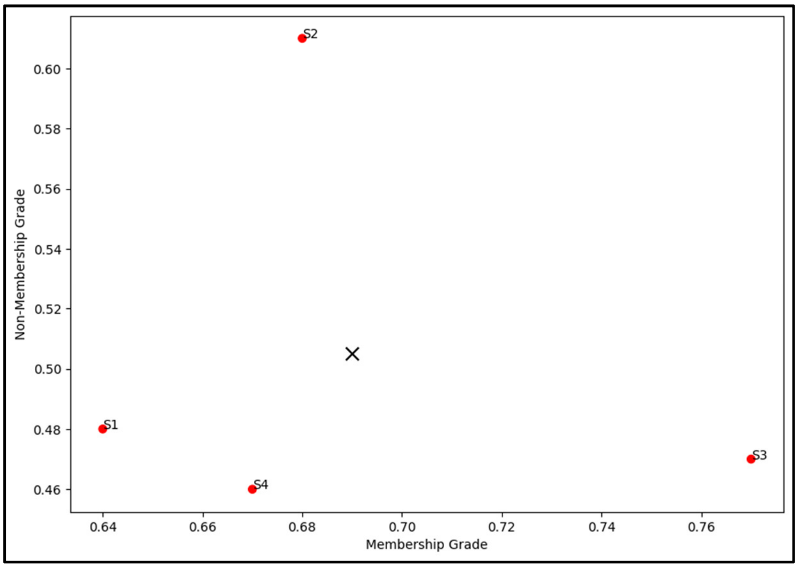


**Fig. 5.** Membership and non-membership grades and cluster centroid of S1 to S4

The optimal design based on our score functions was achieved using the suggested approach. Detailed results are as $S_3 \succ S_4 \succ S_1 \succ S_2$ and best place is $S_3$. Comparative results have been presented in Table 6.

**Table 6**
Comparative results

| Alternatives | Ranks of the alternatives | Best place |
|---|---|---|
| **Our proposed method** | $S_3 \succ S_4 \succ S_1 \succ S_2$ | $S_3$ |
| **Senapati and Yager [11]** | $S_3 \succ S_2 \succ S_1 \succ S_4$ | $S_3$ |
| **Senapati and Yager [10]** | $S_3 \succ S_1 \succ S_4 \succ S_2$ | $S_3$ |

Based on the results suggested by Senapati and Yager [10, 11], the best option was $S_3$, which aligned with our suggested approach. Therefore, it is concluded that our proposed approach provides an alternative method to solve real-life decision-making problems more simply.

## 7. Conclusions

An innovative approach was presented in this research for solving MCDM problems through the aggregation of membership and non-membership values using score functions of Fermatean fuzzy sets. By addressing inherent uncertainties and ambiguities present in complex decision-making scenarios, a significant improvement over the traditional fuzzy set-based approach was offered by this method. The use of a specific score function adjusted for Fermatean fuzzy sets played a crucial role in enhancing the precision and reliability of the aggregation process. The intricate relationships between membership and non-membership values were captured by this function, enabling a more exact representation of diverse criteria.

The effectiveness of the proposed approach was demonstrated through a numerical study in software design, showcasing its practical applicability and potential for optimizing decision-making processes in this domain. The superiority of the proposed method was validated through a comparative study with existing research. The results highlighted the enhanced capability of Fermatean fuzzy sets in managing higher degrees of uncertainty and providing more accurate decision outcomes. This validation underscored the robustness and adaptability of the method, confirming its advantage over traditional MCDM approaches that often fall short in handling complex, real-world problems.

The findings of this study suggest that the proposed aggregation method, grounded in the advanced mathematical framework of Fermatean fuzzy sets, can serve as a valuable tool for decision-makers across various fields. Its application in software design illustrated not only its theoretical robustness but also its practical benefits, paving the way for further research and development in other domains.

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