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Modelling with Neutrosophic Fuzzy Sets for Financial Applications in Discrete System

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ABSTRACT

A unique branch of philosophy known as "Neutrosophy" describes the nature, genesis and scope of neutralities. F. Smarandache originated the neutrosophic set. As an extension of the intuitionistic set, he showed the degree of indeterminacy as an independent component. The application of the neutrosophic numbers, that is, the financial applications in discrete systems is analysed here. In this research paper, we consider the two cases of the financial applications of the neutrosophic environment with its numerical illustration on the basis of fuzzy difference equations.

1. Introduction

The uncertainty theory has a significant impact on how various real-life models are handled in the scientific and engineering fields. Recent years of research on theory and modelling with uncertainty have grown [1-11]. Fuzzy set theory is one of the uncertainties. After the introduction of the fuzzy set by Zadeh [12] and the intuitionistic fuzzy set by Atanassov [13], the ambiguity theories have geared up dramatically. After that, Smarandache [14] proposed the concept of the neutrosophic set. We are already aware that the neutrosophic set considers the truth membership function, the indeterminacy membership function and the falsity membership function simultaneously. Rahaman et al. did the solution of the second-order linear intuitionistic fuzzy difference equation in the paper [15]. This makes them more applicable and productive than the general fuzzy and intuitionistic fuzzy set concepts. In the paper [16], Das et al. showed an application of decision making of a Neutrosophic fuzzy set. A new process was developed with fuzzy clustering based on the association matrix of neutrosophic in the work [17].

Neutrosophic number primarily addresses undetermined, insufficient and inconsistent data. As research continues to execution, we notice that the single-valued neutrosophic set idea is provided by Wang et al. [18], which is the extension part of the neutrosophic set. Ye [19] structured the notion of neutrosophic sets simply and Peng et al. [20,21] presented some basic ideas on these aggregation

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operators with novel operations. Researchers have recently focused on neutrosophic sets and numbers with different kinds of extensions, such as several forms of triangular neutrosophic sets [22], multi-valued neutrosophic sets [23], neutrosophic refined sets [24], interval-valued neutrosophic soft rough sets [25], generalized neutrosophic soft sets [26], single-valued neutrosophic numbers [27], trapezoidal neutrosophic numbers [28], non-linear neutrosophic numbers [29], type-2 neutrosophic numbers [30], etc. Deeba et al. solve a fuzzy difference equation with an application in the research paper [31]. The model of CO2 level in blood is simulated with a fuzzy difference equation and easily processed by Deeba et al. [32] in the work. Lakshmikantham and Vatsala [33] mainly explain the initial theory of fuzzy difference equations. Papaschinopoulos et al. [34] explored the fuzzy difference equation of first-order rational type. Also, in the paper [35], Papaschinopoulos and Stefanidou give an elaborate description of boundedness and the behavior of the solutions asymptotically of a fuzzy difference equation. Umekkan et al. [36] give a financial application of fuzzy difference equations. Zhang et al. [37] provided the behavior of the proper solutions to fuzzy non-linear difference equations. Memarbashi and Ghasemabadi [38] processed the fuzzy difference equations of Volterra form in the work. A fuzzy difference equation of rational type were addressed by Stefanidou and Papaschinopoulos [39] in the study. The application of fuzzy difference equations in finance is also considered and structured by Konstantinos et al. [40] in the research work.

2. Preliminary part

Preliminary mathematical tools are described in this section. The definition and properties of neutrosophic sets are mentioned as follows: Fuzzy sets and their extensions are applied in various real-life applications, including solutions of differential equations [8], series solutions [11], decision-making problems [41-45], mathematical modelling [46-48] and many more.

2.1 Fuzzy set

Consider that, V be a universal set of discourse, where a fuzzy set $\tilde{\mathrm{U}}$ defined on it. Therefore, the fuzzy set $\tilde{\mathrm{U}}$ can be written as [49]:

$$\tilde{\mathrm{U}} = \{(\eta, \mu_{\tilde{\mathrm{U}}}(\eta)) : \eta \in V\} \tag{1}$$

here, the membership function of the fuzzy set $\tilde{\mathrm{U}}$ is $\mu_{\tilde{\mathrm{U}}}(\eta)$ when, $\mu_{\tilde{\mathrm{U}}}: V \to [0,1]$.

2.2 Intuitionistic fuzzy set (IFS): [50,51]

Let us choose that, К be a universal set of discourse, where an intuitionistic fuzzy set $\widetilde{\mathrm{K}}$ defined on this. Then, the intuitionistic fuzzy set $\widetilde{\mathrm{K}}$ is denoted as

$$\widetilde{\mathrm{K}} = \{(\text{þ}, \mu_{\widetilde{\mathrm{K}}}(\text{þ}), \nu_{\widetilde{\mathrm{K}}}(\text{þ})) : \text{þ} \in \mathrm{K}\} \tag{2}$$

where the membership and the non-membership function of the intuitionistic fuzzy set $\widetilde{\mathrm{K}}$ are $\mu_{\widetilde{\mathrm{K}}}(\text{þ})$ and $\nu_{\widetilde{\mathrm{K}}}(\text{þ})$, respectively with $\mu_{\widetilde{\mathrm{K}}}(\text{þ}), \nu_{\widetilde{\mathrm{K}}}(\text{þ}): \mathrm{K} \to [0,1]$ and $0 \le \mu_{\widetilde{\mathrm{K}}}(\text{þ}) + \nu_{\widetilde{\mathrm{K}}}(\text{þ}) \le 1 \; \forall \; \text{þ} \in \mathrm{K}$.

Remark 1: In the intuitionistic fuzzy set (IFS) $\widetilde{\mathrm{K}}$ define in above, if the membership function $(\mu_{\widetilde{\mathrm{K}}}(\text{þ}))$ and non-membership function $(\nu_{\widetilde{\mathrm{K}}}(\text{þ}))$ are satisfies the condition $0 \le \mu_{\widetilde{\mathrm{K}}}(\text{þ}) + \nu_{\widetilde{\mathrm{K}}}(\text{þ}) \le 1$ $\forall$ þ $\in$ К, then the IFS is called a dependable intuitionistic fuzzy set; otherwise undependable intuitionistic fuzzy set.

2.3 Neutrosophic fuzzy set (NFS)

Select that, Ƕ be a universal set of discourse, where a neutrosophic fuzzy set $\widetilde{\mathfrak{W}}$ defined on it. Then, the neutrosophic fuzzy set $\widetilde{\mathfrak{W}}$ is denoted as [52,53]:

$$\widetilde{\mathfrak{W}} = \left\{\left(\varsigma, \mu_{\widetilde{\mathfrak{W}}}(\varsigma), \pi_{\widetilde{\mathfrak{W}}}(\varsigma), \nu_{\widetilde{\mathfrak{W}}}(\varsigma)\right) : \varsigma \in \mathrm{H}\right\} \tag{3}$$

where the membership, indeterminacy membership and non-membership functions of the neutrosophic fuzzy set $(\widetilde{\mathfrak{W}})$ are $\mu_{\widetilde{\mathfrak{W}}}(\varsigma)$, $\pi_{\widetilde{\mathfrak{W}}}(\varsigma)$ and $\nu_{\widetilde{\mathfrak{W}}}(\varsigma)$, respectively with $\mu_{\widetilde{\mathfrak{W}}}(\varsigma), \pi_{\widetilde{\mathfrak{W}}}(\varsigma), \nu_{\widetilde{\mathfrak{W}}}(\varsigma) : \mathrm{H} \rightarrow [0,1]$ and $0 \leq \mu_{\widetilde{\mathfrak{W}}}(\varsigma) + \pi_{\widetilde{\mathfrak{W}}}(\varsigma) + \nu_{\widetilde{\mathfrak{W}}}(\varsigma) \leq 1 \; \forall \, \varsigma \in \mathrm{H}$.

Remark 2: In the neutrosophic fuzzy set (NFS) $\widetilde{\mathfrak{W}}$ define in above, if the membership function $\left(\mu_{\widetilde{\mathfrak{W}}}(\varsigma)\right)$, indeterminacy membership function $\left(\pi_{\widetilde{\mathfrak{W}}}(\varsigma)\right)$ and non-membership function $\left(\nu_{\widetilde{\mathfrak{W}}}(\varsigma)\right)$ are satisfies the condition $0 \leq \mu_{\widetilde{\mathfrak{W}}}(\varsigma) + \pi_{\widetilde{\mathfrak{W}}}(\varsigma) + \nu_{\widetilde{\mathfrak{W}}}(\varsigma) \leq 1 \;\; \forall \, \varsigma \in \mathrm{H}$, then the NFS is called a dependable neutrosophic fuzzy set; otherwise, it is an undependable neutrosophic fuzzy set. In this study, we consider the dependable neutrosophic fuzzy set as a neutrosophic fuzzy set (NFS) for further computation.

2.4 Triangular neutrosophic fuzzy number (TNFN)

Consider that Ƕ be a universal set of discourse, a Triangular Neutrosophic Fuzzy set (TNFS) $\tilde{\mathcal{L}}$ defined on this. Then, the triangular neutrosophic fuzzy set $\tilde{\mathcal{L}}$ is defined as [54]:

$$\tilde{\mathcal{L}} = \left\{\left(\int, \mu_{\tilde{\mathcal{L}}}(\int), \pi_{\tilde{\mathcal{L}}}(\int), \nu_{\tilde{\mathcal{L}}}(\int)\right); (\mathfrak{z}_1, \mathfrak{z}_2, \mathfrak{z}_3; \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3; \mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3) : \int \in \mathrm{H}\right\} \tag{4}$$

where the membership $\left(\mu_{\tilde{\mathcal{L}}}(\int)\right)$, indeterminacy membership $\left(\pi_{\tilde{\mathcal{L}}}(\int)\right)$ and non-membership $\left(\nu_{\tilde{\mathcal{L}}}(\int)\right)$ functions of the neutrosophic fuzzy set $\tilde{\mathcal{L}}$ are represented as

$$\mu_{\tilde{\mathcal{L}}}(\int) = \begin{cases} \frac{\int - \mathfrak{z}_1}{\mathfrak{z}_2 - \mathfrak{z}_1} & ; if \; \mathfrak{z}_1 \leq \int < \mathfrak{z}_2 \\ 1 & ; if \int = \mathfrak{z}_2 \\ \frac{\mathfrak{z}_3 - \int}{\mathfrak{z}_3 - \mathfrak{z}_2} & ; if \; \mathfrak{z}_2 < \int \leq \mathfrak{z}_3 \\ 0 & ; otherwise \end{cases} \tag{5}$$

$$\pi_{\tilde{\mathcal{L}}}(\int) = \begin{cases} \frac{\mathfrak{y}_2 - \int}{\mathfrak{y}_2 - \mathfrak{y}_1} & ; if \; \mathfrak{y}_1 \leq \int < \mathfrak{y}_2 \\ 0 & ; if \int = \mathfrak{y}_2 \\ \frac{\int - \mathfrak{y}_2}{\mathfrak{y}_3 - \mathfrak{y}_2} & ; if \; \mathfrak{y}_2 < \int \leq \mathfrak{y}_3 \\ 1 & ; otherwise \end{cases} \tag{6}$$

and

$$\nu_{\tilde{\mathcal{L}}}(\int) = \begin{cases} \frac{\mathfrak{h}_2 - \int}{\mathfrak{h}_2 - \mathfrak{h}_1} & ; if \; \mathfrak{h}_1 \leq \int < \mathfrak{h}_2 \\ 0 & ; if \int = \mathfrak{h}_2 \\ \frac{\int - \mathfrak{h}_2}{\mathfrak{h}_3 - \mathfrak{h}_2} & ; if \; \mathfrak{h}_2 < \int \leq \mathfrak{h}_3 \\ 1 & ; otherwise \end{cases} \tag{7}$$

where, $0 \leq \mu_{\tilde{\mathcal{L}}}(\int) + \pi_{\tilde{\mathcal{L}}}(\int) + \nu_{\tilde{\mathcal{L}}}(\int) \leq 1 \;\forall \int \in \tilde{\mathcal{L}}$, $\mathfrak{z}_1, \mathfrak{z}_2, \mathfrak{z}_3; \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3; \mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3 \in \tilde{\mathcal{L}}$ and $\mathfrak{z}_1 \leq \mathfrak{z}_2 \leq \mathfrak{z}_3; \mathfrak{y}_1 \leq \mathfrak{y}_2 \leq \mathfrak{y}_3; \mathfrak{h}_1 \leq \mathfrak{h}_2 \leq \mathfrak{h}_3$.

2.5 Parametric form of triangular neutrosophic number: [55]

The parametric form of triangular neutrosophic fuzzy number $(\tilde{\mathcal{L}})$, described by $\tilde{\mathcal{L}} = \{(\int, \mu_{\tilde{\mathcal{L}}}(\int), \pi_{\tilde{\mathcal{L}}}(\int), \nu_{\tilde{\mathcal{L}}}(\int)); (\mathfrak{z}_1, \mathfrak{z}_2, \mathfrak{z}_3; \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3; \mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3): \int \in \text{Ҥ}\}$, it can also said that (α, β, γ) −cut of this triangular neutrosophic fuzzy number $(\tilde{\mathcal{L}})$ is the parametric presentation. Here, in a classical way, it is presented that

$$\begin{aligned} \tilde{\mathcal{L}}_l(\alpha) &= \mathfrak{z}_1 + \alpha(\mathfrak{z}_2 - \mathfrak{z}_1) \\ \tilde{\mathcal{L}}_r(\alpha) &= \mathfrak{z}_3 - \alpha(\mathfrak{z}_3 - \mathfrak{z}_2) \\ \tilde{\mathcal{L}}_l(\beta) &= \mathfrak{y}_2 - \beta(\mathfrak{y}_2 - \mathfrak{y}_1) \\ \tilde{\mathcal{L}}_r(\beta) &= \mathfrak{y}_2 + \beta(\mathfrak{y}_3 - \mathfrak{y}_2) \\ \tilde{\mathcal{L}}_l(\gamma) &= \mathfrak{h}_2 - \gamma(\mathfrak{h}_2 - \mathfrak{h}_1) \\ \tilde{\mathcal{L}}_r(\gamma) &= \mathfrak{h}_2 + \gamma(\mathfrak{h}_3 - \mathfrak{h}_2) \end{aligned} \tag{8}$$

2.6 Example of triangular neutrosophic number

Let us consider that Ҥ be a universal set of discourse, a triangular neutrosophic fuzzy number (TNFN) $\tilde{\mathcal{L}}$ defined on this. Then, the triangular neutrosophic fuzzy number $\tilde{\mathcal{L}}$ is defined as [55]:

$$\tilde{\mathcal{L}} = \{(\int, \mu_{\tilde{\mathcal{L}}}(\int), \pi_{\tilde{\mathcal{L}}}(\int), \nu_{\tilde{\mathcal{L}}}(\int)); (3,5,7;\ 2,5,6;\ 3,5,7): \int \in \mathbb{R}\} \tag{9}$$

where the membership, indeterminacy membership and non-membership function of the neutrosophic fuzzy number $\tilde{\mathcal{L}}$ are $\mu_{\tilde{\mathcal{L}}}(\int)$, $\pi_{\tilde{\mathcal{L}}}(\int)$ and $\nu_{\tilde{\mathcal{L}}}(\int)$ respectively and are denoted as

$$\mu_{\tilde{\mathcal{L}}}(\int) = \begin{cases} \frac{\int-3}{5-3} & ;if\ 3 \le \int < 5 \\ 1 & ;if\ \int = 5 \\ \frac{7-\int}{7-5} & ;if 5 < \int \le 7 \\ 0 & ;otherwise \end{cases} = \begin{cases} \frac{\int-3}{2} & ;if\ 3 \le \int < 5 \\ 1 & ;if\ \int = 5 \\ \frac{7-\int}{2} & ;if\ 5 < \int \le 7 \\ 0 & ;otherwise \end{cases} \tag{10}$$

$$\pi_{\tilde{\mathcal{L}}}(\int) = \begin{cases} \frac{5-\int}{5-2} & ;if\ 2 \le \int < 5 \\ 0 & ;if\ \int = 5 \\ \frac{\int-6}{6-5} & ;if\ 5 < \int \le 6 \\ 1 & ;otherwise \end{cases} = \begin{cases} \frac{5-\int}{3} & ;if\ 2 \le \int < 5 \\ 0 & ;if\ \int = 5 \\ \frac{\int-6}{1} & ;if\ 5 < \int \le 6 \\ 1 & ;otherwise \end{cases} \tag{11}$$

and

$$\nu_{\tilde{\mathcal{L}}}(\int) = \begin{cases} \frac{5-\int}{5-3} & ;if\ 2 \le \int < 5 \\ 0 & ;if\ \int = 5 \\ \frac{\int-7}{7-5} & ;if\ 5 < \int \le 7 \\ 1 & ;otherwise \end{cases} = \begin{cases} \frac{5-\int}{2} & ;if\ 3 \le \int \le 5 \\ 0 & ;if\ \int = 5 \\ \frac{\int-7}{2} & ;if\ 5 < \int \le 7 \\ 1 & ;otherwise \end{cases} \tag{12}$$

where, $(3,5,7;\ 2,5,6;\ 3,5,7) \in \mathbb{R}$ and $3 \le 5 \le 7;\ 2 \le 5 \le 6;\ 3 \le 5 \le 7$. The graphical structure of the triangular neutrosophic fuzzy number (TNFN) $\tilde{\mathcal{L}}$ depicted in Figure 1.

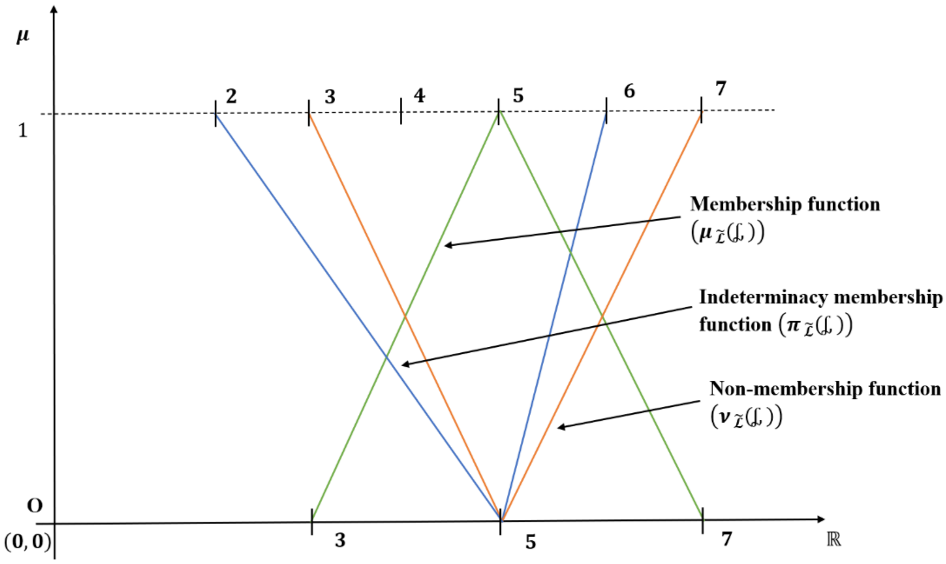

Fig. 1. Geometric representation of a triangular neutrosophic fuzzy number (TNFN) $\tilde{\mathcal{L}}$

Remarks 3: In the fuzzy set with one membership function, namely the true membership function, which only depicts the belongingness of the element in the set and in the intuitionistic fuzzy set has two membership functions, namely the true membership function and non-membership function, which describes the belongingness and non-belongingness of the element in the set. Further, in the neutrosophic fuzzy set, there are three membership functions, namely the membership function, indeterminacy membership function and non-membership function, which represent belongingness, unknown and non-belongingness of the element in the set, respectively. Therefore, the neutrosophic fuzzy describes the element's position more specifically than the fuzzy set and intuitionistic fuzzy set.

3. Difference equation with neutrosophic variable

The form of a q-th order linear difference equation is

$$z_{r+p} = b_1 z_{r+p-1} + b_2 z_{r+p-2} + \cdots + b_p z_r + d_r \tag{13}$$

where $b_1, b_2, \dots, b_p$ and d_r are known constants.

The difference Equation (13) is called the neutrosophic difference equation if

i. All the initial conditions or some of the initial conditions are neutrosophic numbers,
ii. The coefficient or coefficients of the difference Equation (13) are neutrosophic numbers,
iii. The initial conditions and coefficient or coefficients are neutrosophic numbers.

Theorem 1: [56] Let $m \in \mathbb{N}, m \geq 2$. A linear homogeneous system of m first order difference equations are given in matrix form as

$$Z_{r+1} = G Z_r \tag{14}$$

where $Z_r = (Z_r^1, Z_r^2, \dots, Z_r^m)^T$, $G = \left(g_{ij}\right)_{m\times m}, i, j = 1, 2, \dots, m.$

Now, the required solution of Equation (14) is

$$Z_r = G^r Z_0,\ r \in \mathbb{N} \tag{15}$$

Theorem 2: [57] Characterization theorem on neutrosophic number:

Let us choose the neutrosophic initial valued difference equation problem

$$\begin{cases} \tilde{z}_{r+1} = \tilde{f}(z_r, r) \\ \tilde{z}_{r=0} = \tilde{z}_0 \end{cases} \tag{16}$$

where $f: E^* \times \mathbb{Z}_{\geq 0} \to E^*$, i.e.,

i. The parametric form of the function is defined below as follows

$$\left[\tilde{f}\left((z_r, r)\right)\right]_{(\alpha,\beta,\gamma)} = \begin{bmatrix} f_{L,r}^1\left(z_{L,r}^1(\alpha), z_{R,r}^1(\alpha), r, \alpha\right), f_{R,r}^1\left(z_{L,r}^1(\alpha), z_{R,r}^1(\alpha), r, \alpha\right); \\ f_{L,r}^2\left(z_{L,r}^2(\beta), z_{R,r}^2(\beta), r, \beta\right), f_{R,r}^2\left(z_{L,r}^2(\beta), z_{R,r}^2(\beta), r, \beta\right); \\ f_{L,r}^3\left(z_{L,r}^3(\gamma), z_{R,r}^3(\gamma), r, \gamma\right), f_{R,r}^3\left(z_{L,r}^3(\gamma), z_{R,r}^3(\gamma), r, \gamma\right) \end{bmatrix}$$

ii. The functions $f_{L,r}^1\left(z_{L,r}^1(\alpha), z_{R,r}^1(\alpha), r, \alpha\right)$, $f_{R,r}^1\left(z_{L,r}^1(\alpha), z_{R,r}^1(\alpha), r, \alpha\right)$, $f_{L,r}^2\left(z_{L,r}^2(\beta), z_{R,r}^2(\beta), r, \beta\right)$, $f_{R,r}^2\left(z_{L,r}^2(\beta), z_{R,r}^2(\beta), r, \beta\right)$, $f_{L,r}^3\left(z_{L,r}^3(\gamma), z_{R,r}^3(\gamma), r, \gamma\right)$, $f_{R,r}^3\left(z_{L,r}^3(\gamma), z_{R,r}^3(\gamma), r, \gamma\right)$ are considered as continuous functions for any $\in_1 > 0$ $\exists$ a $\delta_1 > 0$, then,

$$\left|f_{L,r}^1\left(z_{L,r}^1(\alpha), z_{R,r}^1(\alpha), r, \alpha\right) - f_{L,r_1}^1\left(z_{L,r_1}^1(\alpha), z_{R,r_1}^1(\alpha), r_1, \alpha\right)\right| < \in_1 \text{ for all } \alpha \in [0,1]$$

with $\left\|\left(z_{L,r}^1(\alpha), z_{R,r}^1(\alpha), r, \alpha\right) - \left(z_{L,r_1}^1(\alpha), z_{R,r_1}^1(\alpha), r_1, \alpha\right)\right\| < \delta_1$ where $r, r_1 \in \mathbb{Z}_{\geq 0}$

and for any $\in_2 > 0$ $\exists$ a $\delta_2 > 0$, then

$$\left|f_{R,r}^1\left(z_{L,r}^1(\alpha), z_{R,r}^1(\alpha), r, \alpha\right) - f_{R,r_2}^1\left(z_{L,r_2}^1(\alpha), z_{R,r_2}^1(\alpha), r_2, \alpha\right)\right| < \in_2 \text{ for all } \alpha \in [0,1]$$

with $\left\|\left(z_{L,r}^1(\alpha), z_{R,r}^1(\alpha), r, \alpha\right) - \left(z_{L,r_2}^1(\alpha), z_{R,r_2}^1(\alpha), r_2, \alpha\right)\right\| < \delta_2$, where $r, r_2 \in \mathbb{Z}_{\geq 0}$.

Similarly, the continuity of the remaining four functions $f_{L,r}^2\left(z_{L,r}^2(\beta), z_{R,r}^2(\beta), r, \beta\right)$, $f_{R,r}^2\left(z_{L,r}^2(\beta), z_{R,r}^2(\beta), r, \beta\right)$, $f_{L,r}^3\left(z_{L,r}^3(\gamma), z_{R,r}^3(\gamma), r, \gamma\right)$, $f_{R,r}^3\left(z_{L,r}^3(\gamma), z_{R,r}^3(\gamma), r, \gamma\right)$ can be defined with this process.

The difference Equation (16) gives to the system of six difference equations described as below

$$\begin{cases} z_{L,r+1}^1(\alpha) = f_{L,r}^1\left(z_{L,r}^1(\alpha), z_{R,r}^1(\alpha), r, \alpha\right) \\ z_{R,r+1}^1(\alpha) = f_{R,r}^1\left(z_{L,r}^1(\alpha), z_{R,r}^1(\alpha), r, \alpha\right) \\ z_{L,r+1}^2(\beta) = f_{L,r}^2\left(z_{L,r}^2(\beta), z_{R,r}^2(\beta), r, \beta\right) \\ z_{R,r+1}^2(\beta) = f_{R,r}^2\left(z_{L,r}^2(\beta), z_{R,r}^2(\beta), r, \beta\right) \\ z_{L,r+1}^3(\gamma) = f_{L,r}^3\left(z_{L,r}^3(\gamma), z_{R,r}^3(\gamma), r, \gamma\right) \\ z_{R,r+1}^3(\gamma) = f_{R,r}^3\left(z_{L,r}^3(\gamma), z_{R,r}^3(\gamma), r, \gamma\right) \end{cases}$$

with the initial conditions $\begin{cases} z_{L,r=0}^1(\alpha) = z_{L,0}^1(\alpha) \\ z_{R,r=0}^1(\alpha) = z_{R,0}^1(\alpha) \\ z_{L,r=0}^1(\beta) = z_{L,0}^1(\beta) \\ z_{R,r=0}^1(\beta) = z_{R,0}^1(\beta) \\ z_{L,r=0}^1(\gamma) = z_{L,0}^1(\gamma) \\ z_{R,r=0}^1(\gamma) = z_{R,0}^1(\gamma) \end{cases}$

Note 1: Every single neutrosophic difference equation is transformed into a system of six crisp difference equations with the help of the characterization theorem. In this article, we have selected only a single dependent variable difference equation in a neutrosophic atmosphere.

Definition 1: [58] Strong and weak solution of neutrosophic difference equation:

The solutions of the difference Equation (16) are described as

a) A strong solution when

$$z_{L,r}^1(\alpha) \leq z_{R,r}^1(\alpha)$$

$$z_{L,r}^{1}(\beta) \leq z_{R,r}^{1}(\beta)$$
$$z_{L,r}^{1}(\gamma) \leq z_{R,r}^{1}(\gamma)$$

and

$$\frac{\partial}{\partial\alpha}\left[z_{L,r}^{1}(\alpha)\right] > 0, \frac{\partial}{\partial\alpha}\left[z_{R,r}^{1}(\alpha)\right] < 0$$
$$\frac{\partial}{\partial\beta}\left[z_{L,r}^{1}(\beta)\right] < 0, \frac{\partial}{\partial\beta}\left[z_{R,r}^{1}(\beta)\right] > 0$$
$$\frac{\partial}{\partial\gamma}\left[z_{L,r}^{1}(\gamma)\right] < 0, \frac{\partial}{\partial\gamma}\left[z_{R,r}^{1}(\gamma)\right] > 0$$

for every $\alpha, \beta, \gamma \epsilon [0,1]$.

b) A weak solution when

$$z_{L,r}^{1}(\alpha) \geq z_{R,r}^{1}(\alpha)$$
$$z_{L,r}^{1}(\beta) \geq z_{R,r}^{1}(\beta)$$
$$z_{L,r}^{1}(\gamma) \geq z_{R,r}^{1}(\gamma)$$

and

$$\frac{\partial}{\partial\alpha}\left[z_{L,r}^{1}(\alpha)\right] < 0, \frac{\partial}{\partial\alpha}\left[z_{R,r}^{1}(\alpha)\right] > 0$$
$$\frac{\partial}{\partial\beta}\left[z_{L,r}^{1}(\beta)\right] > 0, \frac{\partial}{\partial\beta}\left[z_{R,r}^{1}(\beta)\right] < 0$$
$$\frac{\partial}{\partial\gamma}\left[z_{L,r}^{1}(\gamma)\right] > 0, \frac{\partial}{\partial\gamma}\left[z_{R,r}^{1}(\gamma)\right] < 0$$

for every $\alpha, \beta, \gamma \epsilon [0,1]$.

Note 2: Perhaps the strong and weak solution strategy won't happen in certain areas. In this case, the strong and weak solution both exists in the particular time interval or particular interval of α, β or γ. If neither strong nor weak solutions occur, we will call them non-proposed neutrosophic solutions. We recommended taking strong solutions in those cases.

4. Neutrosophic fuzzy sets in financial applications in discrete system

In the research work [59], Kwapisz explores the basic difference equations that explain the balance of a bank deposit. Here, we consider two cases, these are

Case 1. In this case, consider the scenario involving a bank deposit. We know that every bank has a limit for making a stable deposit. Here, the money gets cut when it is deposited under the exact limit. In this paper, E be the cutting rate and R_s be the deposit amount of money. We actually want to detect the final amount of money after deposit it by s years. Now, the difference equation is

$$R_{s+1} = R_s - E \tag{17}$$

where s is the non-negative integer and $E > 0$.

Now, apply the neutrosophic numbers in Equation (17), we get

$$\tilde{R}_{s+1} = \tilde{R}_s - \tilde{E} \tag{18}$$

We take the (α, β, γ)-cuts of the equation (18), i.e.,

$$\left\{\left[{}^{\alpha}_{l}R_{s+1}, {}^{\alpha}_{r}R_{s+1}\right]; \left[{}^{\beta}_{l}R_{s+1}, {}^{\beta}_{r}R_{s+1}\right]; \left[{}^{h}_{l}R_{s+1}, {}^{h}_{r}R_{s+1}\right]\right\}$$
$$= \left\{\left[{}^{\alpha}_{l}R_{s}, {}^{\alpha}_{r}R_{s}\right]; \left[{}^{\beta}_{l}R_{s}, {}^{\beta}_{r}R_{s}\right]; \left[{}^{h}_{l}R_{s}, {}^{h}_{r}R_{s}\right]\right\} - \left\{\left[{}^{\alpha}_{l}E, {}^{\alpha}_{r}E\right]; \left[{}^{\beta}_{l}E, {}^{\beta}_{r}E\right]; \left[{}^{h}_{l}E, {}^{h}_{r}E\right]\right\}$$

Therefore, we have the following system of crisp difference Equation (15) by characterization theorems

$$\begin{cases} {}^{\alpha}_{l}R_{s+1} = {}^{\alpha}_{l}R_{s} - {}^{\alpha}_{r}E \\ {}^{\alpha}_{r}R_{s+1} = {}^{\alpha}_{r}R_{s} - {}^{\alpha}_{l}E \\ {}^{\beta}_{l}R_{s+1} = {}^{\beta}_{l}R_{s} - {}^{\beta}_{r}E \\ {}^{\beta}_{r}R_{s+1} = {}^{\beta}_{r}R_{s} - {}^{\beta}_{l}E \\ {}^{h}_{l}R_{s+1} = {}^{h}_{l}R_{s} - {}^{h}_{r}E \\ {}^{h}_{r}R_{s+1} = {}^{h}_{r}R_{s} - {}^{h}_{l}E \end{cases} \tag{19}$$

Now, the primary balance of the money deposits R_0 be a neutrosophic number and the (α, β, γ) −cut presentation of this is $\left\{[{}^{\alpha}_{l}R_0, {}^{\alpha}_{r}R_0]; \left[{}^{\beta}_{l}R_0, {}^{\beta}_{r}R_0\right]; \left[{}^{\gamma}_{l}R_0, {}^{\gamma}_{r}R_0\right]\right\}$. So, the sequence of the required solution of Equation (19) is

$$\begin{cases} {}^{\alpha}_{l}R_{s} = {}^{\alpha}_{l}R_{0} - s{}^{\alpha}_{r}E \\ {}^{\alpha}_{r}R_{s} = {}^{\alpha}_{r}R_{0} - s{}^{\alpha}_{l}E \\ {}^{\beta}_{l}R_{s} = {}^{\beta}_{l}R_{0} - s{}^{\beta}_{r}E \\ {}^{\beta}_{r}R_{s} = {}^{\beta}_{r}R_{0} - s{}^{\beta}_{l}E \\ {}^{\gamma}_{l}R_{s} = {}^{\gamma}_{l}R_{0} - s{}^{\gamma}_{r}E \\ {}^{\gamma}_{r}R_{s} = {}^{\gamma}_{r}R_{0} - s{}^{\gamma}_{l}E \end{cases} \tag{20}$$

Note 1: The neutrosophic parametric solution of Equation (20) is a divergent solution.

Case 2. This is the case; we show how capitalisation works. Consider that a specific amount of money is deposited in a bank account to get the interest. Here, R_s be the amount of money that is deposited in the bank account to get interest by specific order and $\imath$ be the interest rate. The system is typically used in financial transactions for deposits that accrue interest by years. We actually want to identify the final amount of money after s years of accumulation. Now, the resulting difference equation is

$$R_{s+1} = R_s + \imath R_0 \tag{21}$$

where s be the non-negative integer number.

Consider the Equation (21) in the neutrosophic environment with the initial deposit balance R_0. So,

$$\tilde{R}_{s+1} = \tilde{R}_s - \imath\widetilde{R_0} \tag{22}$$

Let us choose the $(\alpha,\ \beta, \gamma)$ −cuts representation of $\imath$ is $\left\{[{}^{\alpha}_{l}\imath, {}^{\alpha}_{r}\imath]; \left[{}^{\beta}_{l}\imath, {}^{\beta}_{r}\imath\right]; \left[{}^{\gamma}_{l}\imath, {}^{\gamma}_{r}\imath\right]\right\}$. Then, Equation (21) can be indicated into a crisp parametric system of Equation (13), s.t.,

$$\begin{cases} {}^{\alpha}_{l}R_{s+1} = {}^{\alpha}_{l}R_{s} + {}^{\alpha}_{l}\imath{}^{f}_{l}R_0 \\ {}^{\alpha}_{r}R_{s+1} = {}^{\alpha}_{r}R_{s} + {}^{\alpha}_{r}\imath{}^{f}_{r}R_0 \\ {}^{\beta}_{l}R_{s+1} = {}^{\beta}_{l}R_{s} + {}^{\beta}_{l}\imath{}^{\beta}_{l}R_0 \\ {}^{\beta}_{r}R_{s+1} = {}^{\beta}_{r}R_{s} + {}^{\beta}_{r}\imath{}^{\beta}_{r}R_0 \\ {}^{\gamma}_{l}R_{s+1} = {}^{\gamma}_{l}R_{s} + {}^{\gamma}_{l}\imath{}^{\gamma}_{l}R_0 \\ {}^{\gamma}_{r}R_{s+1} = {}^{\gamma}_{r}R_{s} + {}^{\gamma}_{r}\imath{}^{\gamma}_{r}R_0 \end{cases} \tag{23}$$

Solving the system of first order linear non-homogeneous difference equations, which is already mentioned in Equation (23) and the details solution process have been discussed elaborately in [44], we have

$$\begin{cases} {}^{\alpha}_{l}R_s = {}^{\alpha}_{l}R_0 + s{}^{\alpha}_{l}\imath{}^{\alpha}_{l}R_0 \\ {}^{\alpha}_{r}R_s = {}^{\alpha}_{r}R_0 + s{}^{\alpha}_{r}\imath{}^{\alpha}_{r}R_0 \\ {}^{\beta}_{l}R_s = {}^{\beta}_{l}R_0 + s{}^{\beta}_{l}\imath{}^{\beta}_{l}R_0 \\ {}^{\beta}_{r}R_s = {}^{\beta}_{r}R_0 + s{}^{\beta}_{r}\imath{}^{\beta}_{r}R_0 \\ {}^{\gamma}_{l}R_s = {}^{\gamma}_{l}R_0 + s{}^{\gamma}_{l}\imath{}^{\gamma}_{l}R_0 \\ {}^{\gamma}_{r}R_s = {}^{\gamma}_{r}R_0 + s{}^{\gamma}_{r}\imath{}^{\gamma}_{r}R_0 \end{cases} \tag{24}$$

Remarks 4: In this paper, we explore only two cases of financial applications in discrete system under neutrosophic uncertain environment. In this case, we choose a bank deposit related incident exact limitation with specific cutting rate. And, then, we show the work of capitalization with the certain amount of interest rate.

5. Numerical illustration

The numerical example of this proposed model is presented in this section. Numerical illustrations are presented as follows:

Example 1. Choose a situation related to the bank deposit [60]. Each bank has a specific limit for making a permanent deposit. In this problem, the money gets cut when it is deposited under a particular limit. Here, E be the cutting rate and $R_s = 2000€$ be the deposit amount of money. We actually want to identify the amount of money after depositing it in the 3 years. Now, the difference equation is

$$R_{s+1} = R_s - E \tag{25}$$

In Equation (25), $s = 3$ and $E > 0$.

Now, apply the neutrosophic numbers in Equation (25), we get

$$\tilde{R}_{s+1} = \tilde{R}_s - \tilde{E} \tag{26}$$

where $\tilde{E} = \{0.022, 0.023, 0.024;\ 0.021, 0.022, 0.023;\ 0.023, 0.024, 0.025\}$ and $\tilde{R}_0 = 2000€$ is the fixed amount.

We take the (α, β, γ) −cuts of the equation (26), i.e.,

$$\left\{[{}^{\alpha}_{l}R_{s+1}, {}^{\alpha}_{r}R_{s+1}]; \left[{}^{\beta}_{l}R_{s+1}, {}^{\beta}_{r}R_{s+1}\right]; [{}^{h}_{l}R_{s+1}, {}^{h}_{r}R_{s+1}]\right\} = \left\{[{}^{\alpha}_{l}R_s, {}^{\alpha}_{r}R_s]; \left[{}^{\beta}_{l}R_s, {}^{\beta}_{r}R_s\right]; [{}^{h}_{l}R_s, {}^{h}_{r}R_s]\right\} - \left\{[{}^{\alpha}_{l}E, {}^{\alpha}_{r}E]; \left[{}^{\beta}_{l}E, {}^{\beta}_{r}E\right]; [{}^{h}_{l}E, {}^{h}_{r}E]\right\}$$

Now, if the initial balance of the money deposits R_0 be a neutrosophic number and the (α, β, γ) −cuts presentation of $\tilde{E}$ is $\{[0.022 + 0.001\alpha, 0.024 - 0.001\alpha\ ]; [0.022 - 0.001\beta, 0.022 + 0.001\beta]; [0.024 - 0.001\gamma,\ 0.024 + 0.001\gamma]\}$. So, the sequence of the required solution is,

$$\begin{cases} {}^{\alpha}_{l}R_s = 2000 - s(0.024 - 0.001\alpha) \\ {}^{\alpha}_{r}R_s = 2000 - s(0.022 + 0.001\alpha) \\ {}^{\beta}_{l}R_s = 2000 - s(0.022 + 0.001\beta) \\ {}^{\beta}_{r}R_s = 2000 - s(0.022 - 0.001\beta) \\ {}^{\gamma}_{l}R_s = 2000 - s(0.024 + 0.001\gamma) \\ {}^{\gamma}_{r}R_s = 2000 - s(0.024 - 0.001\gamma) \end{cases}$$

Here, $\tilde{R}_0$ is the fixed value and then, we put, $s = 0,\ 0.2,\ 0.4,\ 0.6,\ 0.8, 1, \ldots, 3$ which is a crisp number and find the required solution of it and $0 < (\alpha,\ \beta, \gamma) \leq 1$.

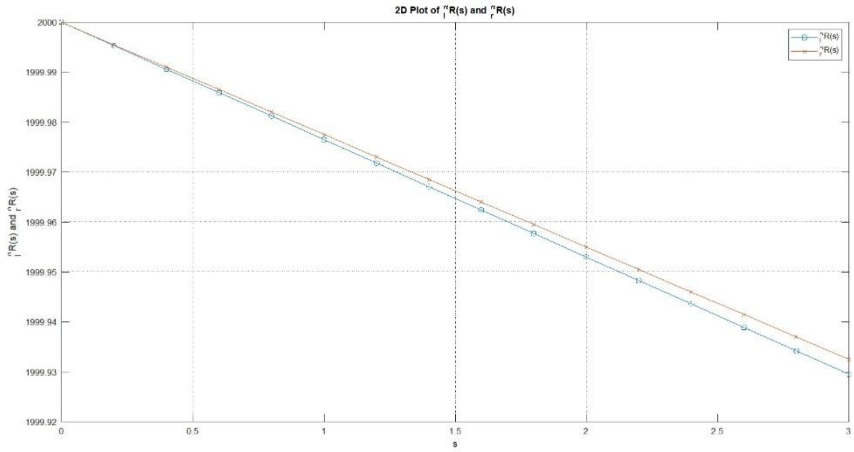

Fig. 2. Membership curve of solutions of Example 1 at $\alpha = 0.5$ in 2D

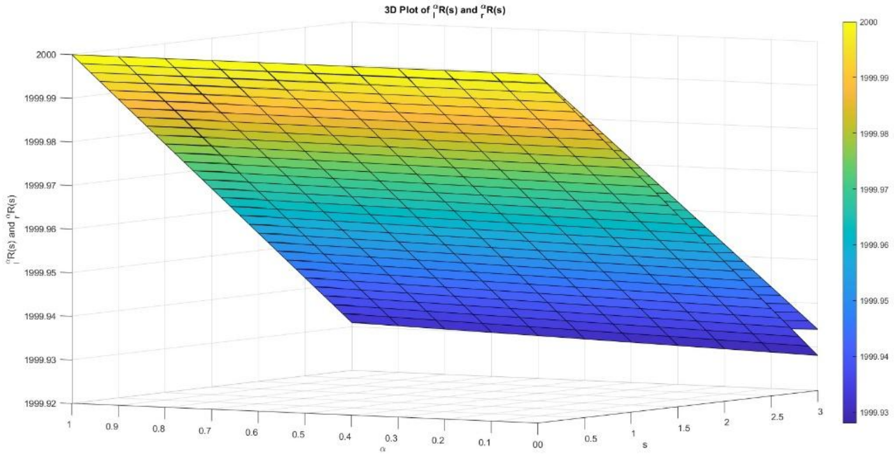

Fig. 3. Membership curve of solutions of Example 1 at $\alpha \in [0,1]$ in 3D

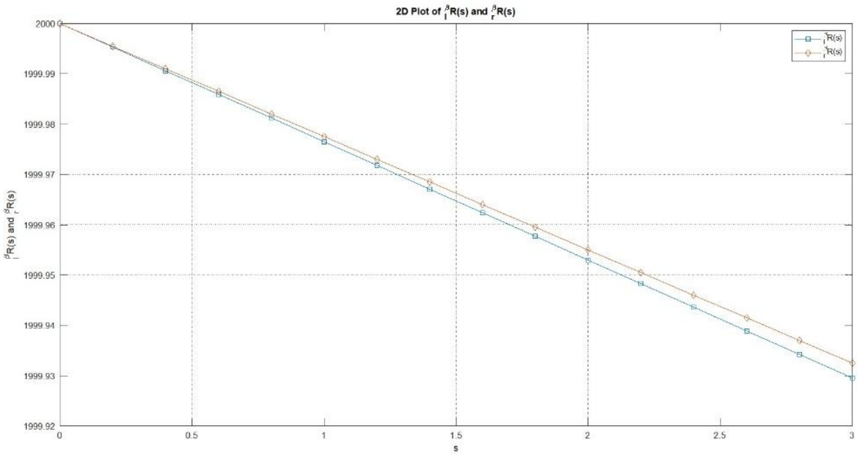

Fig. 4. Indeterminacy membership curve of solutions of Example 1 at $\beta = 0.5$ in 2D

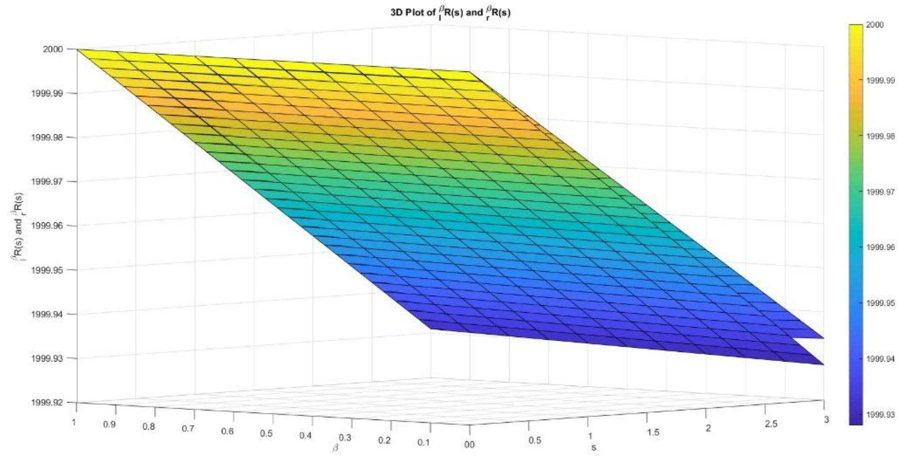

Fig. 5. Indeterminacy membership curve of solutions of Example 1 at $\beta \in [0,1]$ in 3D

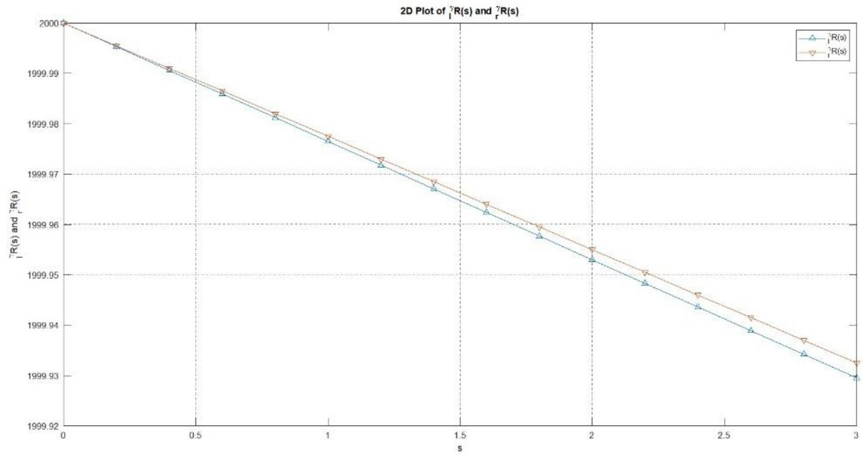

Fig. 6. Non-membership curve of solutions of Example 1 at $\gamma = 0.5$ in 2D

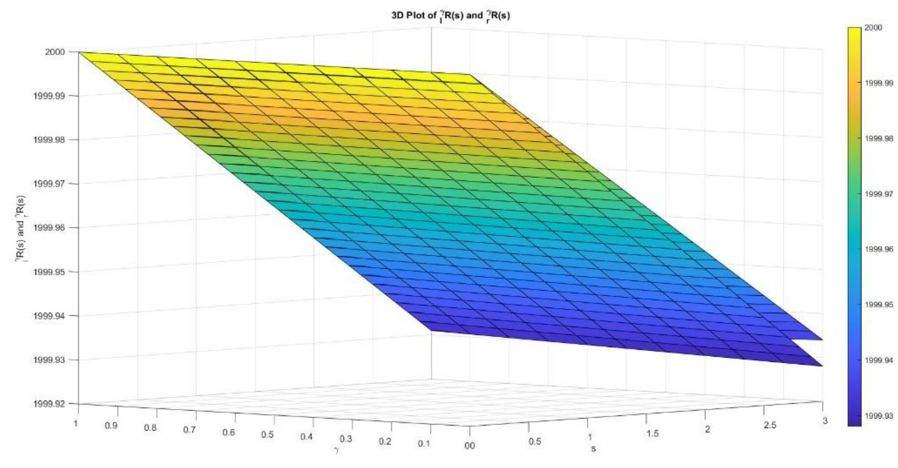

Fig. 7. Non-membership curve of solutions of Example 1 at $\gamma \in [0,1]$ in 3D

Figure 2 and Figure 3 represent the solution space of the membership function of Example 1 in two and three dimensional spaces, respectively. Similarly, Figure 4 and Figure 5 represent the solution space of the indeterminacy membership function of Example 1 in two and three dimensional spaces, respectively. Further, Figure 6 and Figure 7 represent the solution space of the non-membership function of Example 1 in two and three dimensional paces, respectively. All the solution spaces are triangular in form.

Example 2. Select that the specific amount of money is deposited in a bank account to get the interest [61]. Here, R_s be the order to get interest and $\imath = \{0.022, 0.023, 0.024;\ 0.021, 0.022, 0.023;\ 0.023, 0.024, 0.025\}$ be the interest rate. We define the final amount of money after 3 years of accumulation. Now, the resulting difference equation is

$$R_{s+1} = R_s + \imath R_0 \tag{27}$$

where s be the crisp number.

Consider Equation (27) in the neutrosophic environment with the initial deposit balance R_0. So

$$\tilde{R}_{s+1} = \tilde{R}_s - \imath\widetilde{R_0} \tag{28}$$

Let us choose the (α, β, γ) −cuts representation of $\imath$ is $\{[0.022 + 0.001\alpha, 0.024 - 0.001\alpha\ ]; [0.022 - 0.001\beta, 0.022 + 0.001\beta]; [0.024 - 0.001\gamma,\ 0.024 + 0.001\gamma]\}$ and $\tilde{R}_0 = 2000€$ is the fixed value of money. Then, after indicating the crisp parametric systems of Equation (28), we solve the system of first order linear non-homogeneous difference equations and the detailed solution of this is

$$\begin{cases} {}_l^{\alpha}R_s = 2000 - \imath(0.022 + 0.001\alpha)2000 \\ {}_r^{\alpha}R_s = 2000 - \imath(0.024 - 0.001\alpha)2000 \\ {}_l^{\beta}R_s = 2000 - \imath(0.022 - 0.001\beta)2000 \\ {}_r^{\beta}R_s = 2000 - \imath(0.022 + 0.001\beta)2000 \\ {}_l^{\gamma}R_s = 2000 - \imath(0.024 - 0.001\gamma)2000 \\ {}_r^{\gamma}R_s = 2000 - \imath(0.024 + 0.001\gamma)2000 \end{cases} = \begin{cases} {}_l^{\alpha}R_s = 2000 - \imath(44 + 2\alpha) \\ {}_r^{\alpha}R_s = 2000 - \imath(48 - 2\alpha) \\ {}_l^{\beta}R_s = 2000 - \imath(44 - 2\beta) \\ {}_r^{\beta}R_s = 2000 - \imath(44 + 2\beta) \\ {}_l^{\gamma}R_s = 2000 - \imath(48 - 2\gamma) \\ {}_r^{\gamma}R_s = 2000 - \imath(48 + 2\gamma) \end{cases}$$

Now, we put, $\imath = 0,\ 0.2,\ 0.4,\ 0.6,\ 0.8, 1, \ldots, 3$ which is a crisp number to get the required solution of it and $0 < (\alpha,\ \beta, \gamma) \leq 1$.

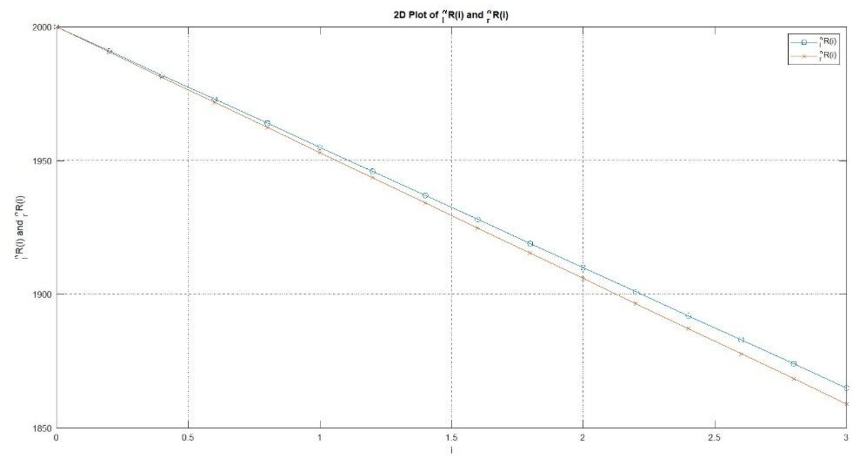

Fig. 8. Membership curve of solutions of Example 2 at $\alpha = 0.5$ in 2D

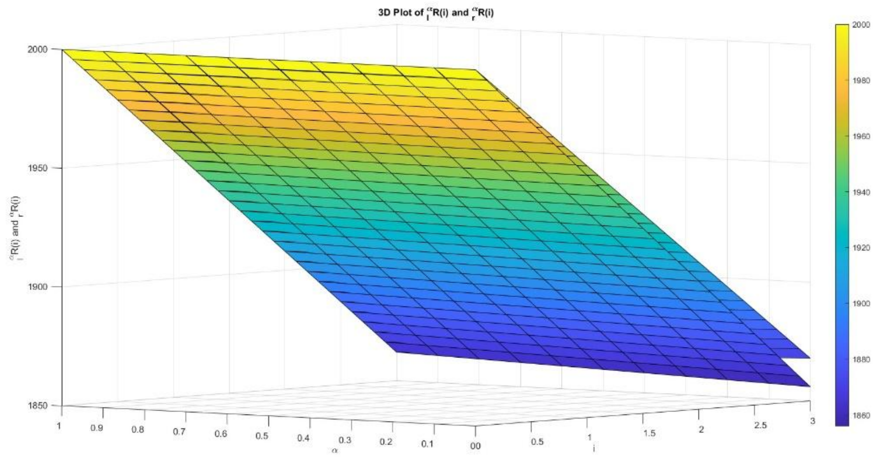

Fig. 9. Membership curve of solutions of Example 2 at $\alpha \in [0,1]$ in 3D

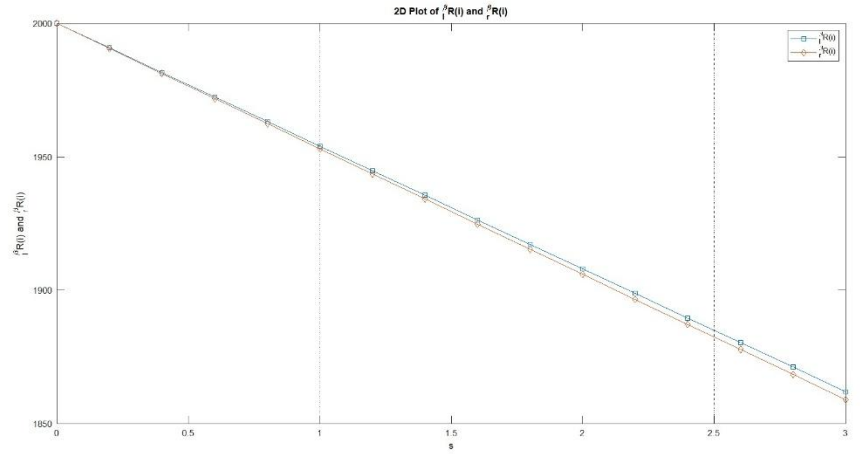

Fig. 10. Indeterminacy membership curve of solutions of Example 2 at $\beta = 0.5$ in 2D

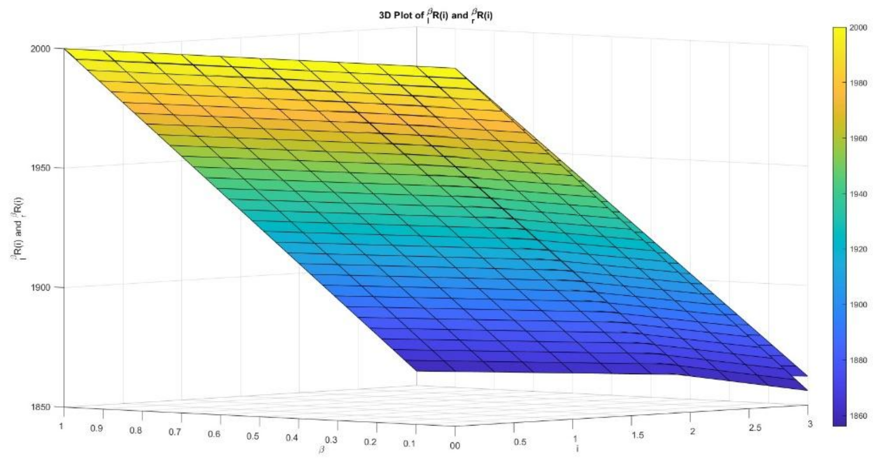

Fig. 11. Indeterminacy membership curve of solutions of Example 2 at $\beta \in [0,1]$ in 3D

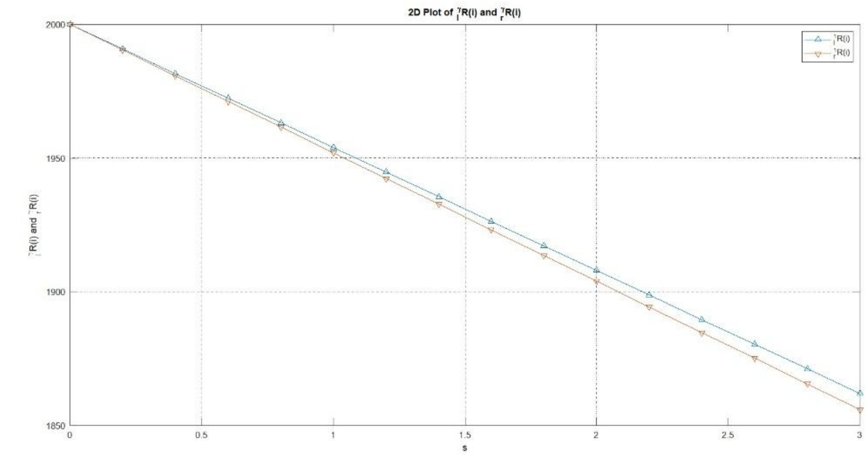

Fig. 12. Non-membership curve of solutions of Example 2 at $\gamma = 1$ in 2D

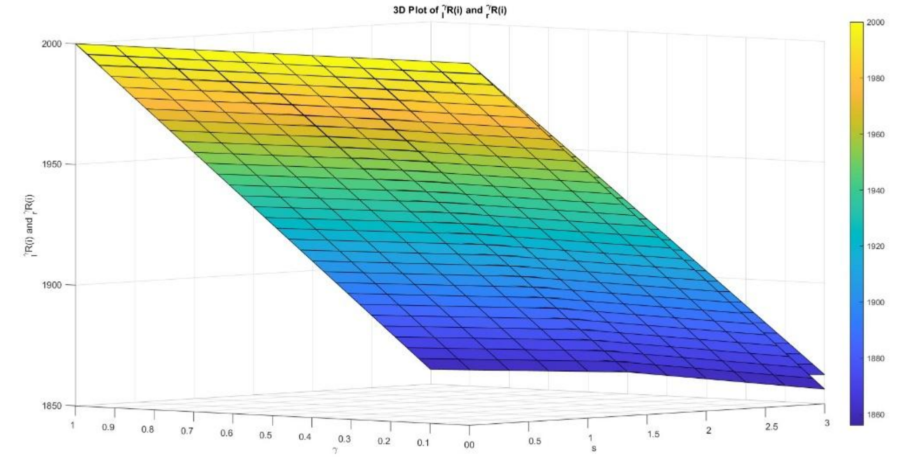

Fig. 13. Non-membership curve of solutions of Example 2 at $\gamma \in [0,1]$ in 3D

Figure 8 and Figure 9 represent the solution space of the membership function of Example 2 in two and three dimensional spaces, respectively. Similarly, Figure 10 and Figure 11 represent the solution space of the indeterminacy membership function of Example 2 in two and three dimensional spaces, respectively. Further, Figure 12 and Figure 13 represent the solution space of the non-membership function of Example 2 in two and three dimensional paces, respectively. All the solution spaces are triangular in form.

6. Conclusions and future research scope

The Neutrosophic fuzzy sets in financial modelling convey a powerful framework for handling the uncertainty and ambiguity of data in discrete systems. Neutrosophic sets account for three degrees of membership, i.e., truth, uncertainty and falsehood, which helps in making more nuanced decisions. This methodology is particularly effective in handling situations such as banking problems, stock market analysis, portfolio optimization, etc. In this paper, the fuzzy difference equation used in the neutrosophic environment helps to increase the predictive accuracy and reliability of financial models. In addition, neutrosophic fuzzy systems facilitate the growth of adaptive algorithms capable of responding to real-world financial environments. In this research work, we consider two different banking related problems in a fuzzy neutrosophic environment with the help of basic difference equations.

In this section, the future research outlines on this topic are described. This work can be extended with different cases of financial applications in discrete systems. Anybody can apply a different fuzzy environment to process the application. In the numerical illustration section, real-life banking related problems can be further explored with the help of mathematical examples. In a word, it can be said that, to enhance further performance, it can focus on refining computational processes and analysing the hybrid models that combine neutrosophic sets with other machine learning methods to further boost performance in the difficult field of financial systems.

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Data Availability Statement
There is no data for this research.

Conflicts of Interest
The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper and the funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, or in the decision to publish the results.

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